



# FAI Sporting Code

*Fédération  
Aéronautique  
Internationale*

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## **Section 7F – XC Scoring CIVL GAP Centralised Cross-Country Competition Scoring for Hang Gliders and Paragliders Classes 1 to 5**

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Editor's note: Hang-gliding and paragliding are sports in which both men and women participate. Throughout this document the words "he", "him" or "his" are intended to apply equally to either sex unless it is specifically stated otherwise.

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1	FAI Statutes,	Chapter 1,	para. 1.6	
2	FAI Sporting Code, Gen. Section,		Chapter 4,	para 4.1.2
3	FAI Statutes,	Chapter 1,	para 1.8.1	
4	FAI Statutes,	Chapter 2,	para 2.1.1; 2.4.2; 2.5.2 and 2.7.2	
5	FAI By-Laws,	Chapter 1,	para 1.2.1	
6	FAI Statutes,	Chapter 2,	para 2.4.2.2.5	
7	FAI By-Laws,	Chapter 1,	paras 1.2.2 to 1.2.5	
8	FAI Statutes,	Chapter 5,	paras 5.1.1, 5.2, 5.2.3 and 5.2.3.3	
9	FAI Sporting Code, Gen. Section,		Chapter 4,	para 4.1.5
10	FAI Sporting Code, Gen. Section,		Chapter 2,	para 2.2.
11	FAI Statutes,	Chapter 5,	para 5.2.3.3.7	
12	FAI Statutes,	Chapter 6,	para 6.1.2.1.3	

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# 1 Introduction

This document contains all definitions required to score centralised cross-country competitions for both hang-gliding and paragliding. Its main purpose is to serve as an addendum to sections 7A and 7B of the FAI sporting code. Additionally, it should serve as an educational tool for all parties involved in such competitions, as a reference for the implementation of scoring systems, as well as a basis for future improvements and modifications.

## 1.1 Scope

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The document's scope is restricted to scoring of FAI Category 1 cross-country competitions for hang-gliding and paragliding: World and Continental championships in both sports. CIVL's rule setting targets these competitions exclusively, whereas organisers of FAI Category 2 competitions as well as non-sanctioned competitions are free to score their competitions however they like. Most of them do follow CIVL's lead, though, so this document should also cover the majority of Category 2 events as well.

## 1.2 Sources

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The remainder of this document is based on:

"The GAP Guide" (2011 edition)

FAI Sporting Code Section 7A for Hang-Gliding (2013 edition)

FAI Sporting Code Section 7B for Paragliding (2013 edition)

The scoring implementation within CIVL's scoring software, FS (aka "FScomp")

Appendix C of the Paragliding World Cup Association's 2013 Competition Rules

## 1.3 Changes from previous edition

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Changes from the previous edition of this document are marked in **red color**. If the section title is marked in red it means the whole section text was added or changed.

## 1.4 Differences between Hang-Gliding and Paragliding

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Initially, both hang-gliding and paragliding competitions used the same system for scoring, generally known as "GAP". But over time, through their two separate sporting codes, the two disciplines introduced more and more changes that would only apply to one, but not the other. This mainly in non-standard situations such as stopped tasks, pilots landing just short of goal, or pilots crossing the start line too early. Where not explicitly stated otherwise, the contents of this document always apply to both disciplines. Definitions applying only to one, but not the other, are clearly marked as such.



Text marked in blue applies exclusively to hang-gliding.



Text marked in orange applies exclusively to paragliding.

## 2 The GAP Philosophy

CIVL's scoring system is generally known as "GAP", named after the first-name initials of its three inventors Gerolf Heinrichs (G), Angelo Crapanzano (A) and Paul Mollison (P). Their intention was to "create a fair scoring system easily adaptable to any competition anywhere in the world, both for hang gliding and paragliding, with a philosophy that is easy for the pilot to understand, regardless of the mathematical complexity of the underlying formulas".

### 2.1 History

Work on GAP started in 1998, and it was officially introduced in 2000, to allow scoring of competitions based on GPS track logs, instead of photographic evidence as it had been used until then.

#### 2002

An updated version, named "GAP 2002" was published. This introduced the concept of leading points, which are calculated by comparing the complete track logs of all pilots in a task. Leading points replaced the departure points used in GAP 2000

#### 2005

A variation of GAP 2002 was introduced in Australia, named "OzGAP" or "OzGAP 2005". The difference to GAP lies mainly in the way arrival points are calculated, but this was never adopted by CIVL.

#### 2008

"GAP 2008" was officially released. The main scoring mechanisms remained unchanged from the 2002 edition, but the implementation of GAP 2008 included several rules introduced in the sporting codes for either hang-gliding or paragliding. These cover stopped tasks, starting too early, and landing between the end of the speed section and goal.

#### 2011

"GAP 2011" marked another software release where the main scoring remained unchanged from the 2002 definition and implementation. The main changes were all for paragliding: altitude bonus in stopped tasks, as well as a reduced number of available points in stopped tasks and in tasks with no pilots in goal.

#### 2012

The "Jump the Gun" rule for early starts in hang-gliding competitions changed in S7A. This was implemented in FS, but this was also released, unfortunately, as "GAP 2011".

#### 2014

The edition introduced several significant changes for paragliding, a few of which also applied to hang gliding. Most of those changes originated from the Paragliding World Cup Association (PWCA), and 2014 was the first time that both CIVL and PWCA scored their paragliding competitions using the same formula. The changes were: Nominal launch parameter, see 5.1(hang gliding and paragliding)

1. Final glide decelerator(paragliding)
2. Goal shape, see 6.3 (paragliding)
3. Purely linear distance points, see 11.1 (paragliding)
4. Adjusted formula for leading points, see 11.3 (paragliding)
5. No more arrival points, see 11.4 (paragliding)
6. Scoring of stopped tasks, see 12.3 (hang gliding and paragliding)
7. Use of FTV for competition scoring, see 15(paragliding)

#### 2015

This 2015 edition applies the 2014 changes in leading points calculations for paragliding now also to hang gliding. In paragliding, the use of final glide deceleration methods is no longer mandatory. QNH is now used as the primary altitude measurement. Distance calculations continue to be based on the FAI sphere, the introduction of a new distance measurement regime, based on the WGS84 ellipsoid, has been postponed.

#### 2016

In Paragliding, the leading points weight (maximum available leading points) is doubled compared to the previous version. This reduces the available time points. Also, if no pilot is in goal, the weight is now calculated as the ratio between task distance and actual distance covered by the pilot who flew the furthest. The maximum in this case is 0.1 (equivalent to 100 points for a task with task quality 1).

In hang gliding, the penalty for jumping the gun was increased for one point every three seconds to one point every 2 seconds.

**2018**

Distance measurement for paragliding is based on the WGS84 ellipsoid. For FAI Category 1 events, a lower turnpoint radius tolerance is chosen. In hang-gliding, the goal line is now also replaced by a semi-circle facing away from the previous turnpoint.

**2020**

The changes are:

1. Remove final glide decelerators
2. No more prescribed turnpoint direction (including start)
3. Clarify task distance calculation
4. Clarify rule for restarts for races with multiple start gates and for elapsed time tasks
5. Use a constant leading weight for paragliding
6. Task results are given with one decimal point, only round once when calculating competition results
7. Adopt the PWCA's leading points calculation for paragliding
8. Adopt the PWCA's time points calculation for paragliding and hang-gliding
9. Minimum time for stopped tasks depends on Nominal Time
10. Stopped tasks: Redistribute removed time points as distance points
11. FTV: Use best score for FTV validity

## **2.2 Scoring Process**

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Scoring follows a nine-step process, as depicted in Figure 1:

Setting the competition parameters, or "GAP parameters", according to the competition site, the expected pilot level and the expected tasks. This happens once for each competition, at the outset, and must not be changed throughout the competition. See section 4.

Setting a task – this happens typically once per day on flyable days. See section 6.

Letting the pilots fly the task. See section 7.

Evaluating the task, by collecting all pilots' track logs for this task, and determining for each pilot the distance flown and, if the end of speed section was reached, in what time this happened. See section 8.

Calculating the task validity based on the task's statistical values such as fastest time to ESS, number of pilots in goal, average distance flown and several others. See section 9.

Points allocation: Calculating the maximum number of points awarded for distance, speed, leading and arrival, based on the task validity and the statistical values found in the task evaluation. See section 10.

Scoring each pilot's flight, by calculating the awarded points for distance, speed, leading and arrival. The outcome, the pilots' total score, is the sum of these four values. See sections 11 and 12.

Ranking all pilots according to their total score for the task results. See section 13.

Aggregation of task results for competition scoring and ranking. See section 14.

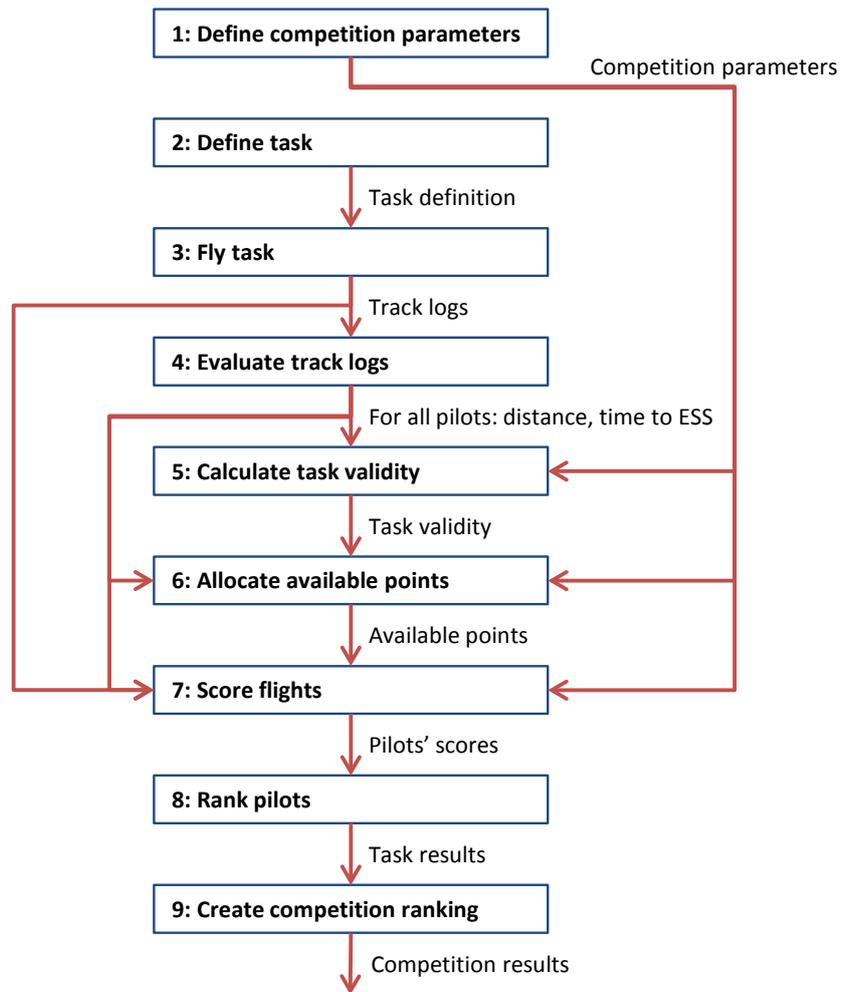


Figure 1: Scoring process

### 3 Definitions

The definitions of flights, locations, distances and times of CIVL GAP are described in Section 7A 5.2.1.

Flight

Free flight

Competition task

Competition flight

Take-off

Speed section

Start of speed section (SSS)

Turnpoint (TP)

Control zone

End of speed section (ESS).

Goal

Landing place

Task distance

Flown distance

Finish point

Race start

Start time

Start gate

Window open time

Task deadline

Finish time

Task time

Landing time

## 4 Use of Tracklog Data

### 4.1 Position

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Coordinates of positions, such as turn points or pilot positions, are always given as WGS84 coordinates, based on the WGS84 ellipsoid. The coordinate format is UTM by default, but other formats can be chosen by organisers as appropriate.

### 4.2 Distance

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In general, task evaluation occurs in the x/y plain, therefore distance measurements are always exclusively horizontal measurements. All distances are calculated on the WGS84 ellipsoid.

For altitude bonus in stopped tasks (12.3.6), altitude is also considered, but this does not affect distance calculations between two geographic points.

### 4.3 Altitude

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All altitude evaluation is primarily based on barometric altitude, as recorded in the flight instrument tracklog (the International Standard Atmosphere pressure altitude QNE) and then when necessary corrected by the scoring software for the pressure conditions of the task (QNH). GNSS altitude may be taken into consideration (from the primary tracklog or a backup log) only in case of problems with barometric logging.

Category 2 Organisers may choose to use the less accurate **GNSS** altitude instead of barometric altitude.

### 4.4 Time

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Time evaluation is based on **UTC** time, as given in GPS tracklogs. For better readability, times of the day may be expressed in local time for the competition location.

## 5 Competition Parameters

Before the first task, the following parameters must be defined by the meet director, or another person or group as defined by the competition's local regulations:

Nominal Launch

Nominal Distance

Minimum Distance

Nominal Goal

Nominal Time

The values set for these parameters define how each task's validity is calculated. They should therefore be chosen very carefully, considering the realistic potential of the flying site. Setting the values too low will prevent the formula from distinguishing between demanding, high quality tasks and quick, easy low quality tasks which are sometimes the only option due to weather conditions.

In paragliding competitions, the following must also be defined by the same person or group who defines the first five competition parameters above:

Score-back time for stopped task

### 5.1 Nominal Launch

When pilots do not take off for safety reasons, to avoid difficult launch conditions or bad conditions in the air, Launch Validity is reduced (see section 9.1). Nominal Launch defines a threshold as a percentage of the pilots in a competition. Launch Validity is only reduced if fewer pilots than defined by that threshold decide to launch.

The recommended default value for Nominal Launch is 96%, which means that Launch Validity will only be reduced if fewer than 96% of the pilots present at launch chose to launch.

### 5.2 Nominal Distance

Nominal distance should be set to the expected average task distance for the competition. Depending on the other competition parameters and the distances actually flown by pilots, tasks shorter than Nominal Distance will be devalued in most cases. Tasks longer than nominal distance will usually not be devalued, as long as the pilots fly most of the distance.

In order for GAP to be able to distinguish between good and not-so-good tasks, and devalue the latter, it is important to set nominal distance high enough<sup>13</sup>.

### 5.3 Minimum Distance

The minimum distance awarded to every pilot who takes off. It is the distance below which it is pointless to measure a pilot's performance. The minimum distance parameter is set so that pilots who are about to "bomb out" will not be tempted to fly into the next field to get past a group of pilots – they all receive the same amount of points anyway.

### 5.4 Nominal Goal

The percentage of pilots the meet director would wish to have in goal in a well-chosen task. This is typically 20 to 40%. This parameter has a very marginal effect on distance validity (see section 9.2).

<sup>13</sup> See also this excellent series of articles on the subject: Part 1:<http://ozreport.com/1360767307>; Part 2:<http://ozreport.com/1360858575>; Part 3: <http://ozreport.com/1360944246>

## **5.5 Nominal Time**

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Nominal time indicates the expected task duration, the amount of time required to fly the speed section. If the fastest pilot's time is below nominal time, the task will be devalued. There is no devaluation if the fastest pilot's time is above nominal time.

Nominal time should be set to the expected "normal" task duration for the competition site, and nominal distance / nominal time should be a bit higher than typical average speeds for the area.

## **5.6 Score-back Time**

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In a stopped task, this value defines the amount of time before the task stop was announced that will not be considered for scoring. The default is 5 minutes, but depending on local meteorological circumstances, it may be set to a longer period for a whole competition. See also section 12.3.1.

## 6 Task Setting

### 6.1 Definition of a task

A task can be either a race task or an open distance task.

#### 6.1.1 Race task

A race task definition consists of:

- A launch point, given as **WGS84** coordinates
- A number of control zones
- A goal
- An indication which of the control zones is the start (start of speed section)
- If goal does not serve as end of speed section: An indication which of the control zones is the end of speed section
- A launch time window
- A start procedure, including timing
- Optionally, a task deadline

In exceptional circumstances, with regard to restricted launch areas and poor flying conditions, to ensure the task is fair for 2/3rds of the pilots, a task may be run without leading/departure points. This is to be declared at the task briefing.

#### 6.1.2 Open distance task

An open distance task definition consists of:

- A launch point, given as **WGS84** coordinates
- A number of control zones
- Optionally, an indication which of the control zones is the start
- Optionally, a direction for the final, open distance leg
- A launch time window
- If a start control zone exists: A start time
- A task deadline

### 6.2 Definition of control zones

Control zones are geographical areas which must be reached by the pilots in the course of a task. The types of control zones are the turnpoint cylinder and the goal.

#### 6.2.1 Turnpoint cylinder

A turnpoint cylinder is defined as:

- A centre point  $c$ , given as **WGS84** coordinates
- A radius  $r$ , given in meters

A turnpoint cylinder is then given as the cylinder with radius  $r$  around the axis which cuts the  $x/y$  plain orthogonally at the cylinder's centre point  $c$ . For task evaluation purposes, only the cylinder's projection in the  $x/y$  plain is considered: a circle of radius  $r$  around  $c$ .

Note that the designation of "enter" or "exit" cylinder has been removed, to reduce a potential source of confusion and task setting errors. Whether a turnpoint is considered reached is determined either by the presence of a single tracklog point inside the turnpoint cylinder's tolerance band, or by the presence of two

consecutive tracklog points which lie on opposite sides of the turnpoint cylinder boundary (a “crossing”). The direction in which such a crossing occurs is irrelevant.

Task setters may still choose to indicate whether the start or subsequent turnpoint cylinders are “enter” or “exit”, to explain their intended task route. But pilots are not bound to those indications.

## 6.2.2 Definition of goal

A goal can be

- A **turnpoint** cylinder or
- A goal line, defined by:
  - A centre point  $c$ , given as **WGS84** coordinates
  - A length  $l$ , given in meters
  - A **previous point  $p$** , given as **WGS84** coordinates

### 6.2.2.1 Goal line

The goal **line** control zone consists of the semi-circle with radius  $l/2$  behind the goal line, when coming from the last turn point that is different from the goal line centre. Entering that zone without prior crossing of the goal line is equivalent to crossing the goal line.

Physical lines can be used in addition to the official, virtual goal line as defined by **WGS84** coordinates, to increase attractiveness for spectators and media, and to increase visibility for pilots. Physical lines must be at least 50m long and 1m wide, made of white material and securely attached to the ground. The physical line must match as closely as possible the corresponding virtual line as defined by the goal GPS coordinates and the direction of the last task leg. It must not be laid out further from the previous turn point than the goal GPS coordinates.

By default, the goal line length  $l$  is set to 400m.

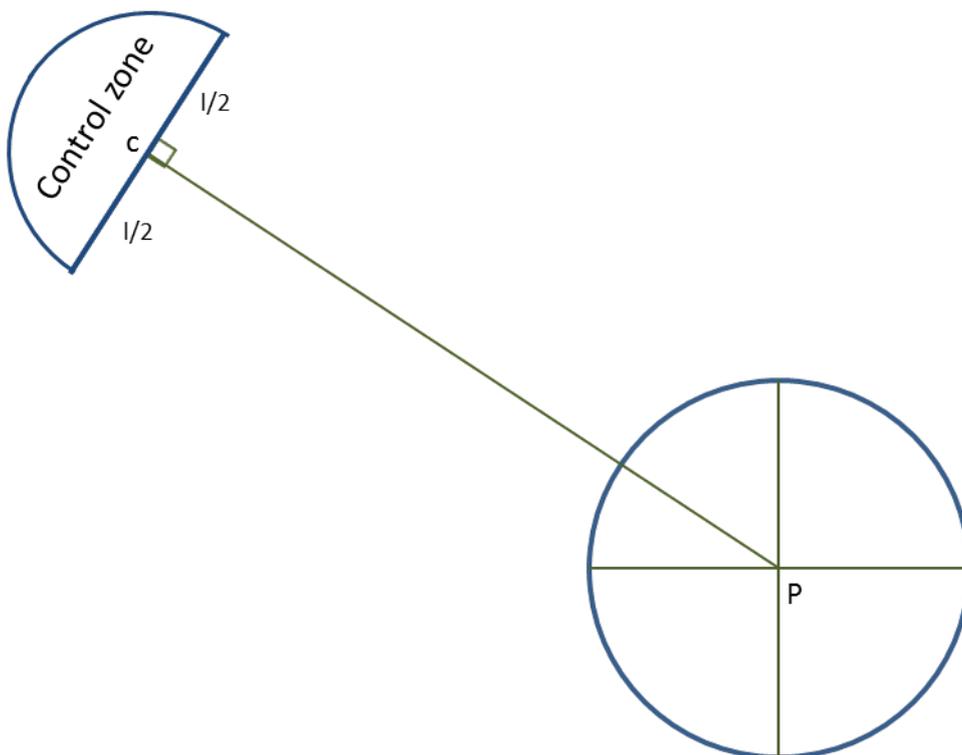


Figure 2: Goal line

## 6.3 Start procedures

Start procedures define how an individual pilot’s start time is determined. A start can be either air- or ground-started, and it can be either a race to goal or an elapsed time start.

### 6.3.1 Air start

For air-started tasks, the competitors are free to launch any time during the launch window, and to fly about, regardless of any turnpoint or start cylinders, up to their race start. Race start is defined as the crossing of the start cylinder in the prescribed direction for the last time before continuing to fly through the remainder of the task.

### 6.3.2 Ground start

In a ground-started task, the race starts with the pilots' launch. Since a launch can be difficult to detect from **anIGC tracklog**, the task setters must set a cylinder around the launch area as the first turnpoint. A pilot's start is registered when he exits this cylinder for the first time. In the case of a race to goal task, the launch window open time is the same as the first (or only) start gate time.

### 6.3.3 Race to goal

A race to goal start is defined by one or more so-called "start gates". The first – or only – start gate is given as a daytime. Subsequent start gates are given as a time interval, along with the number of start gates.

Example 1: "We have a Race to Goal task; the start gate opens at 13:00"

Example 2: "We have a Race to Goal task with 5 start gates from 13:30 at a 20 minutes interval." – the start gate times in this case are 13:30, 13:50, 14:10, 14:30, and 14:50.

Pilots are free to start any time after the first (or single) start gate. A pilot's start time is then defined as the time of the last start gate after which he started flying the speed section of the task.

Example 3: Given the start gates from Example 2 above, pilot A, crossing the start cylinder at 13:49:01, will be given a start time of 13:30. Any pilot starting after 14:50 will be given a start time of 14:50.

Starting before the first (or only) start gate is considered a failed start. Some refer to this as "jumping the gun". The two disciplines handle failed starts differently, see section 12.2.

### 6.3.4 Elapsed time

An elapsed time start is defined by a single "start gate", given as a daytime. Pilots are free to start any time after this start gate. A pilot's start time is then defined as the time at which he started flying the speed section of the task. Each pilot has therefore an individual start time.

Example 1: "We have an Elapsed Time task, the start gate opens at 12:30" – pilot A starting at 12:31:03 has a start time of 12:31:03, pilot B starting at 15:48:28 has a start time of 15:48:28.

## 6.4 Distances

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### 6.4.1 Route optimization algorithm

At the core of all distance calculations lies an algorithm that calculates the shortest path from a given start position through a series of turnpoint cylinders to a destination (which can be a cylinder, a goal line or a point). The algorithm used to determine this path is defined in Appendix A. The result of this algorithm is a set of points on the cylinders that represent the optimal crossing points for shortest overall distance. Distance is then calculated by iterating through those points, calculating the distance between two subsequent points (leg distance), and building the sum of those leg distances.

*routeElement* = point(*lat, lon*)|cylinder(*lat, lon, radius*)|goal(*lat, lon, radius*)

*routeDefinition* = {*routeElement*<sub>1</sub>, ..., *routeElement*<sub>*n*</sub>}

*path* = {*point*<sub>1</sub>, ..., *point*<sub>*n*</sub>} where *point*<sub>*x*</sub> = point(*lat*<sub>*x*</sub>, *lon*<sub>*x*</sub>)

getShortestPath(*routeDefinition*) returns *path*

pathDistance(*path*) =  $\sum_{i=1}^{n-1} \text{distance}(\text{path.point}_i, \text{path.point}_{i+1})$

## 6.4.2 Task distances

Task distance is defined as speed section distance plus any distance pilots need to cover before or after the speed section:

$$\begin{aligned}
 \mathit{launch} &= \text{point}(\mathit{lat}_{\mathit{launch}}, \mathit{lon}_{\mathit{launch}}) \\
 \mathit{taskPart1} &= \left\{ \begin{array}{l} \mathit{launch}, \\ \mathit{preTurnpoint}_1, \dots, \mathit{preTurnpoint}_m, \\ \mathit{startOfSpeedSection}, \\ \mathit{turnpoint}_1, \dots, \mathit{turnpoint}_n, \\ \mathit{endOfSpeedSection} \end{array} \right\} \\
 \mathit{taskPart1Path} &= \text{getShortestPath}(\mathit{taskPart1}) \\
 \mathit{taskPart1Distance} &= \text{pathDistance}(\mathit{taskPart1Path}) \\
 \mathit{preSpeedSectionDistance} &= \text{pathDistance}(\mathit{taskPart1Path}[0.. \mathit{startOfSpeedSectionIndex}]) \\
 \mathit{speedSectionDistance} &= \mathit{taskPart1Distance} - \mathit{preSpeedSectionDistance} \\
 \mathit{postSpeedSectionDefinition} &= \left\{ \begin{array}{l} \mathit{taskPart1Path.point}_{\mathit{last}}, \\ \mathit{postTurnpoint}_1, \dots, \mathit{postTurnpoint}_n, \\ \mathit{goal} \end{array} \right\} \\
 \mathit{postSpeedSectionPath} &= \text{getShortestPath}(\mathit{postSpeedSectionDefinition}) \\
 \mathit{postSpeedSectionDistance} &= \text{pathDistance}(\mathit{postSpeedSectionPath}) \\
 \mathit{taskDistance} &= \left( \begin{array}{l} \mathit{preSpeedSectionDistance} \\ + \mathit{speedSectionDistance} \\ + \mathit{postSpeedSectionDistance} \end{array} \right)
 \end{aligned}$$

## 7 Flying a task

There are 2 task types: race task and open distance task. The definitions are given in Section 7A-5.2.2.

### 7.1 Re-starting

In tasks that are either set as Race to Goal with multiple start gates, or as Elapsed Time task, a pilot may decide to return to the start and take a later start time after already having flown a portion of the **speed section**.

~~A restart is possible up to the point where the first turnpoint has been reached. Returning to the start after reaching the first turnpoint has no effect on the scored start time.~~

## 8 Task evaluation

From each pilot's track, task evaluation determines the distance this pilot flew along the task, and the time this pilot took to fly the speed section.

### 8.1 Cylinder tolerance

To compensate for the very slight distance measurement differences resulting from the use of different distance measurement algorithms in flight recorders and evaluation programs, a tolerance, consisting of a percentage and an absolute minimum, is applied when determining whether a pilot reached a cylinder. This had to be introduced so that a pilot reading the distance to the next cylinder centre from his flight instrument can rely on having reached the turnpoint when the distance displayed by the instrument is smaller than the defined turnpoint cylinder radius.

For Cat 1, the tolerance is set to 0.1%.

For Cat 2, the maximum tolerance is 0.5%, to allow pilots to still use equipment that calculates distances on the FAI sphere.

$$tolerance = 0.1\% \mid 0.5\%$$

$$minTolerance = 5m$$

$$turnpoint_i : innerRadius_i = \min(radius_i * (1 - tolerance), radius_i - minTolerance)$$

$$turnpoint_i : outerRadius_i = \max(radius_i * (1 + tolerance), radius_i + minTolerance)$$

### 8.2 Reaching a cylinder

To determine whether a pilot reached a specific cylinder in the task, the pilot's track log must show evidence that:

1. The pilot reached the previous cylinder in the task definition
2. It contains at least one valid crossing for the cylinder, defined as a single tracklog point inside the turnpoint cylinder's tolerance band, which was recorded
  - a. after the valid crossing of the previous cylinder in the task definition
  - b. not earlier than the start time
  - c. no later than the task deadline

$$\forall i : \exists turnpoint_i :$$

$$crossings_i = \forall j : distance(turnpoint_i.center, trackpoint_j) \geq innerRadius_i$$

$$\wedge distance(turnpoint_i.center, trackpoint_j) \leq outerRadius_i : trackpoint_j$$

Crossing time for each valid crossing is the time at which the corresponding tracklog point was recorded.

$$crossing_{i,j}.time = trackpoint_j.time$$

Finally, given all  $n$  crossings for a turnpoint cylinder, sorted in ascending order by their crossing time, the time when the cylinder was reached is determined.

In Race to Goal tasks with single start gate, the reaching time for a turnpoint is always the first crossing time after the start gate time, and after the reaching time of the previous turnpoint.

$$reachingTime_i = crossing_{i,n}.time :$$

$$n = \min(m) : crossing_m.time > reachingTime_{i-1}$$

In Race to Goal tasks with multiple start gates, and in Elapsed Time tasks, the reaching time for SSS is that turnpoint's last crossing time that is before all the subsequent turnpoint's reaching times. The reaching time for any turnpoint after SSS is that turnpoint's first crossing time after the previous turnpoint's reaching time.

$turnpoint_i = SSS : reachingTime_i = crossing_{i,n}.time :$

$n = \max(m) : crossing_{i,m}.time < reachingTime_{i+1}$

$turnpoint_i \neq SSS : reachingTime_i = crossing_{i,n}.time :$

$n = \min(m) : crossing_{i,m}.time > reachingTime_{i-1}$

### 8.2.1 Start time

In Race to Goal tasks the start time for pilots who reached SSS after the first start gate time is equal to the last start gate time before their SSS reaching time.

$\forall p : p \in PilotsFlown \wedge \exists reachingTime_{p,SSS} : startTime_p = SSS.gateTime_n :$

$n = \max(m) : SSS.gateTime_m < reachingTime_{p,SSS}$

In Elapsed Time tasks, the pilot's start time is equal to their reaching time of SSS.

$\forall p : p \in PilotsFlown \wedge \exists reachingTime_{p,SSS} : startTime_p = reachingTime_{p,SSS}$

## 8.3 Reaching goal

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### 8.3.1 Goal cylinder

Verification of a pilot reaching a goal cylinder is achieved by the same method as verification of reaching a turnpoint cylinder (8.2)

### 8.3.2 Goal line

To reach goal in the case of a goal line, the goal line must be crossed in flight. This is achieved when a line drawn between two adjacent points in the pilot's tracklog crosses the goal line in the correct direction.

Entering the goal control zone (semi-circle behind the goal line, see 6.2.2.1) from any direction without prior crossing of the goal line is equivalent to crossing the goal line. **To determine whether the goal control zone was reached, the same tolerance calculations apply as for full cylinders (see 8.1).**

If a physical line is used, crossing either the virtual or the physical goal line counts as having reached goal. An official observation (through a goal marshal or similar) of a pilot crossing the line in flight overrules a negative goal crossing decision based on the pilot's tracklog. Not crossing a physical goal line for obvious safety reasons must be considered in the pilots' favour.

 The physical goal line is crossed when the hang glider's nose cuts the line, in the correct direction, before a landing is made.

 The physical goal line is crossed when the paraglider pilot's leading foot cuts the line, in the correct direction, before a landing is made.

## 8.4 Flown distance

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### 8.4.1 Race task

To determine a pilot's flown distance, a first step determines which turnpoints he reached considering all timing restrictions: launch within launch time window, valid start, but only until the task deadline time. After the last turnpoint the pilot reached, for every remaining track point, the shortest distance to goal is calculated using the method described in section 6.5.1. The flown distance is then calculated as task distance minus the shortest distance the pilot still had to fly. Therefore, for scoring, the pilot's best distance along the course line is considered, regardless of where the pilot landed in the end.

If a pilot flies less than minimum distance, he will be scored for this minimum distance. This also applies to pilots who are not able to produce a valid tracklog, but for whom launch officials verify launch within the launch window.

If a pilot reaches goal, he will be scored for the task distance.

$$\forall p : p \in \text{PilotsLandingBeforeGoal} \text{ bestDistance}_p = \max(\text{minimumDistance}, \text{taskDistance} - \min(\forall \text{track}_p.\text{point}_i, \text{shortestDistanceToGoal}(\text{track}_p.\text{point}_i)))$$

$$\forall p : p \in \text{PilotsReachingGoal} \text{ bestDistance}_p = \text{taskDistance}$$

### 8.4.2 Open distance task

In an open distance task, if a pilot lands before the last of any given control zones, his flown distance is calculated according to 8.4.1. For pilots flying further than the last control zone, the flown distance is calculated by adding the pilot's best distance flown in the open distance part of the task (after the last control zone) to task distance between launch and the last control zone. If the task gave a direction for the open distance leg, then the pilot's best distance projected onto that direction is considered.

## 8.5 Time for speed section

---

The time a pilot took to fly the speed section is determined by his start time (which is influenced by the task's start procedure and the time he crossed the start of speed section cylinder) and the time when he crossed the end of speed section after reaching all previous turn points. The smallest unit for time measurement is one second.

Pilots who do not reach the end of speed section cylinder do not get a time.

$$\forall p : p \in \text{PilotsReachingESS} : \text{time}_p = \text{timeAtESS}_p - \text{startTime}_p$$

## 9 Task Validity

The task validity is a value between 0 and 1 and measures how suitable a competition task is to evaluate pilots' skills. It is calculated for each task after the task has been flown, by multiplying the three validity coefficients: Launch validity, distance validity, and time validity.

$$\text{TaskValidity} = \text{LaunchValidity} * \text{DistanceValidity} * \text{TimeValidity}$$

### 9.1 Launch Validity

Launch validity depends on nominal launch and the percentage of pilots actually present at take-off who launched. If the percentage of pilots on take-off who launch is equal to nominal launch, or higher, then launch validity is 1. If, for example, only 20% of the pilots present at take-off launch, launch validity goes down to about 0.1.

The reasoning behind launch validity: Launch conditions may be dangerous, or otherwise unfavourable. If a significant number of pilots at launch think that the day is not worth the risk of launching, then the gung-ho pilots who did go will not get so many points. This is a safety mechanism.

'Pilots present' are pilots arriving on take-off, with their gear, with the intention of flying. For scoring purposes, 'Pilots present' are all pilots not in the 'Absent' status (ABS): Pilots who took off, plus pilots present who did not fly (DNF). DNFs need to be attributed carefully. A pilot who does not launch due to illness, for instance, is not a DNF, but an ABS.

$$\text{LVR} = \min\left(1, \frac{\text{NumberOfPilotsFlying}}{\text{NumberOfPilotsPresent} * \text{NominalLaunch}}\right)$$

$$\text{LaunchValidity} = 0.027 * \text{LVR} + 2.917 * \text{LVR}^2 - 1.944 * \text{LVR}^3$$

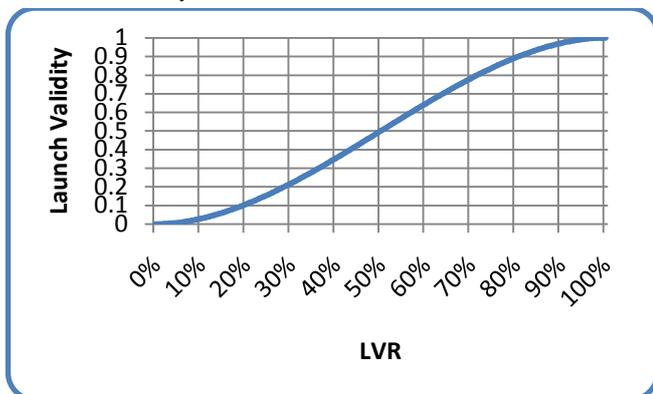


Figure 3: Launch validity curve

### 9.2 Distance Validity

Distance validity depends on nominal distance, nominal goal, the longest distance flown and the sum of all distances flown beyond minimum distance. If the task distance is quite short in relation to nominal distance, the day is probably not a good measure of pilot skill because there would not be many decisions to make.

If a task is longer than nominal distance, the day will not be devalued because of distance validity, even if the nominal goal parameter value is not achieved, as long as a fair percentage of pilots fly a good distance. This sounds like a vague statement, but the task setter should try to set tasks that are reasonable for the day and achievable. If everyone lands in goal, you must ask if this was a valid test of skill - it probably was if the fastest time and the distance flown were reasonably long. If everyone lands short of goal, was it an unsuitable task but still a good test of pilot skill? You also can have the case where a task that is shorter than nominal distance, has a distance validity of almost 1. This will happen when a large percentage of the pilots fly a large percentage of the course but, in this case, you still have a practical devaluation because there will be little spreading between pilots' scores.

In the formula below, 'p' denotes an individual pilot.

$$SumOfFlownDistancesOverMinDist = \sum_p \max(0, FlownDist_p - MinDist)$$

$$NomDistArea = \frac{((NomGoal + 1) * (NomDist - MinDist)) + \max(0, (NomGoal * (BestDist - NomDist)))}{2}$$

$$DVR = \frac{SumOfFlownDistancesOverMinDist}{NumPilotsFlying * NomDistArea}$$

$$DistanceValidity = \min(1, DVR)$$

### 9.3 Time Validity

Time validity depends on the fastest time to complete the speed section, in relation to nominal time. If the fastest time to complete the speed section is longer than nominal time, then time validity is always equal to 1.

If the fastest time is quite short, the day is probably not a good measure of pilot skill because there would not be many decisions to make and, because of this, luck can distort scores as there will be little possibility to recover any accidental loss of time.

If no pilot finishes the speed section, then time validity is not based on time but on distance: The distance of the pilot who flies the furthest in relation to nominal distance is then used to calculate the time validity the same way as if it was the time.

If one pilot reached ESS:  $TVR = \min(1, \frac{BestTime}{NominalTime})$

If no pilot reached ESS:  $TVR = \min(1, \frac{BestDistance}{NominalDistance})$

$$TimeValidity = \max(0, \min(1, -0.271 + 2.912 * TVR - 2.098 * TVR^2 + 0.457 * TVR^3))$$

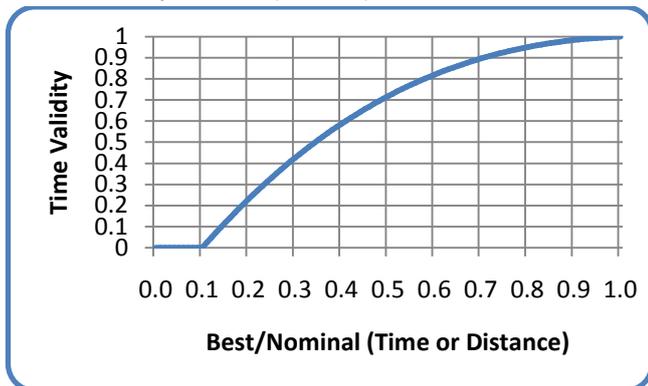


Figure 4: Time validity curve

## 10 Points Allocation

The available points for each task are 1000\*Task Validity. These points are distributed between distance points, time points, leading points and arrival points. The distribution depends on the percentage of pilots who reached goal before the task deadline, compared to pilots who launched, as well as the chosen goal form. It is expressed in terms of weight factors for each of the four point categories: Distance weight, time weight, leading weight and arrival weight. Weight factors are always between 0 and 1. A weight factor of 0.5 for distance, for example, means that 50% of the day's available overall points are available for distance points.

$$GoalRatio = \frac{NumberOfPilotsInGoal}{NumberOfPilotsFlying}$$

$$DistanceWeight = 0.9 - 1.665 * GoalRatio + 1.713 * GoalRatio^2 - 0.587 * GoalRatio^3$$

$$LeadingWeight = \frac{1 - DistanceWeight}{8} * 1.4$$

$$ArrivalWeight = \frac{1 - DistanceWeight}{8}$$

$$GoalRatio = 0: DistanceWeight = 0.838$$

$$GoalRatio > 0: DistanceWeight = 0.805 - 1.374 * GoalRatio + 1.413 * GoalRatio^2 - 0.484 * GoalRatio^3$$

$$LeadingWeight = 0.162$$

$$ArrivalWeight = 0$$

$$TimeWeight = 1 - DistanceWeight - LeadingWeight - ArrivalWeight$$

$$AvailableDistancePoints = 1000 * TaskValidity * DistanceWeight$$

$$AvailableTimePoints = 1000 * TaskValidity * TimeWeight$$

$$AvailableLeadingPoints = 1000 * TaskValidity * LeadingWeight$$

$$AvailableArrivalPoints = 1000 * TaskValidity * ArrivalWeight$$

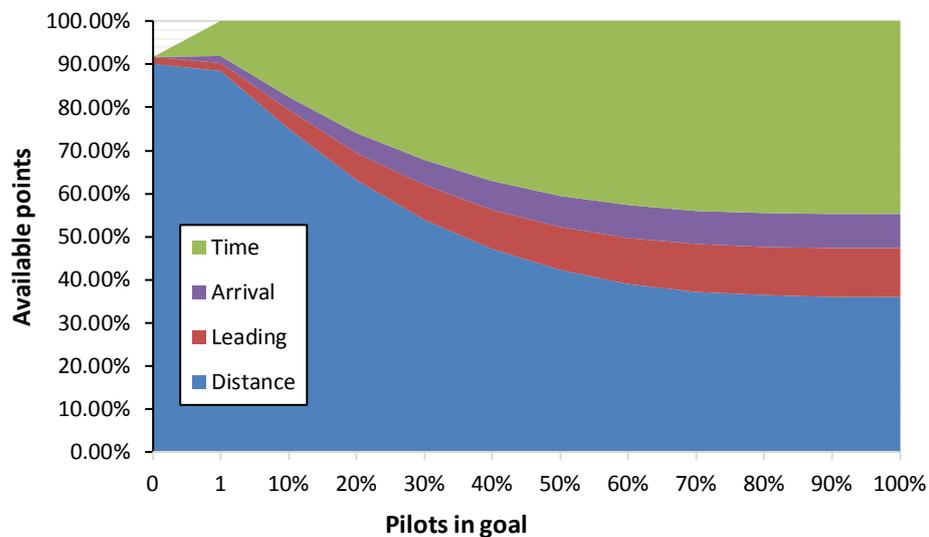


Figure 5: Points allocation for hang gliding

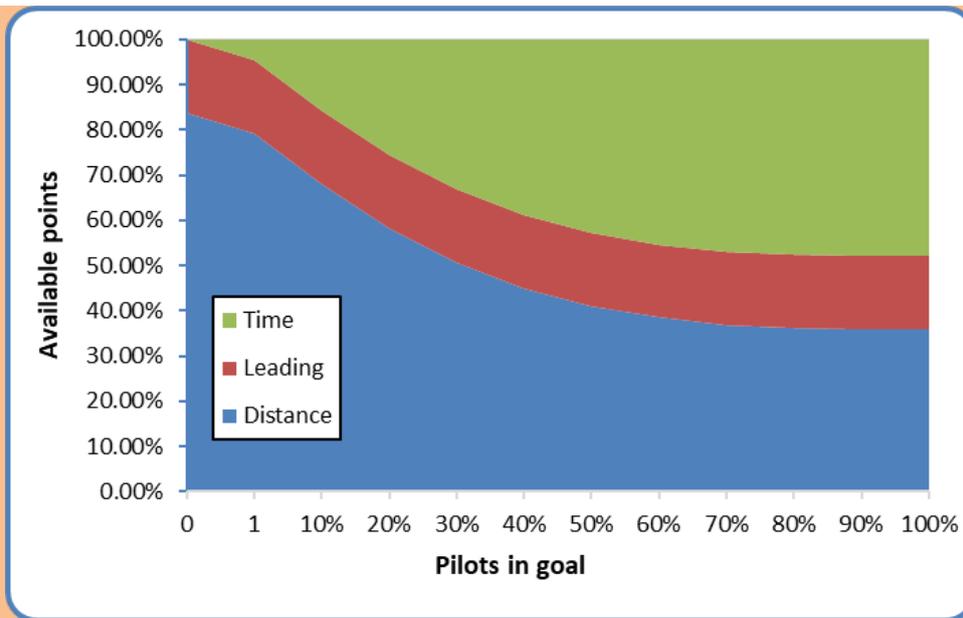


Figure 6: Points allocation for paragliding

From the above it follows that in hang-gliding, if nobody reaches ESS, then a maximum of 900 points are available for distance and 18 points for leading but, of course, no points for time nor arrival. This is also the maximum possible number of points for an open distance task.

$numberOfPilotsAtESS = 0$  :

$AvailableDistancePoints = 1000 * TaskValidity * DistanceWeight$

$AvailableTimePoints = 0$

$AvailableLeadingPoints = 1000 * TaskValidity * LeadingWeight$

$AvailableArrivalPoints = 0$

$Max(AvailableDistancePoints) = 900$

$Max(availableLeadingPoints) = 18$

$Max(availableTotalPoints) = 918$

## 11 Pilot score

Each pilot's score is the sum of that pilot's distance, time, leading and arrival points, **rounded to one decimal place**

$$\forall p: p \in \text{PilotsLaunched} : \text{TotalScore}_p = \text{DistancePoints}_p + \text{TimePoints}_p + \text{LeadingPoints}_p + \text{ArrivalPoints}_p$$

### 11.1 Distance points

The distance considered for each pilot to calculate distance points is that pilot's best distance along the course line, up until the pilot landed or the task deadline was reached, whichever comes first.

 One half of the available distance points are assigned to each pilot linearly, based on the pilot's distance flown in relation to the best distance flown in the task. The other half is assigned taking into consideration the difficulty of the kilometers flown.

$$\begin{aligned} \text{LinearFraction}_p &= \frac{\text{Distance}_p}{2 * \text{BestDistance}} \\ \text{iDist10}_p &= \text{int}(\text{Distance}_p * 10) \\ \text{DifficultyFraction}_p &= \text{DiffScore}_{\text{iDist10}_p} + ((\text{DiffScore}_{\text{iDist10}_p+1} - \text{DiffScore}_{\text{iDist10}_p}) \\ &* (\text{Distance}_p * 10 - \text{iDist10}_p)) \\ \text{DistancePoints}_p &= (\text{LinearFraction}_p + \text{DifficultyFraction}_p) * \text{AvailableDistancePoints} \end{aligned}$$

 The available distance points are assigned to each pilot linearly, based on the pilot's distance flown in relation to the best distance flown in the task.

$$\text{DistancePoints}_p = \frac{\text{Distance}_p}{\text{BestDistance}} * \text{AvailableDistancePoints}$$

In the case of a stopped task, a pilot's distance may be increased by an altitude bonus (see 12.3.2).

#### 11.1.1 Difficulty calculation

 To measure the relative difficulty of each 100 meters of the task, we consider the number of pilots who landed in the successive few kilometers, and the distance flown.

In a first step, for each 100-meter section of the task, the number of pilots who landed in that section is counted. Pilots who landed before minimum distance are counted as having landed at minimum distance. Only pilots who landed out are considered for this calculation, pilots who reached goal are not counted.

$$\forall i: i < \text{int}(\text{MinDist} * 10) : \text{PilotsLanded}_i = 0$$

$$\text{PilotsLanded}_{\text{int}(\text{MinDist} * 10)} = \sum_{\forall \text{Pilot}: \text{int}(\text{Pilot.Distance} * 10) \leq \text{int}(\text{MinDist} * 10)} 1$$

$$\forall i: i > \text{int}(\text{MinDist} * 10) \& i \leq \text{int}(\text{MaxDist} * 10) : \text{PilotsLanded}_i = \sum_{\forall q: q \in \text{PilotsLandedOut}: \text{int}(\text{Distance}_q * 10) = i} 1$$

Then the difficulty for each 100-meter section of the task is calculated by counting the number of pilots who landed further along the task. If 100 pilots land out on a flight of 100 km, the next 3 km are considered. If 10 pilots land out in 100 km, the next 30 km are considered. The variable LookAheadDist contains the number of 100-meter slots to look ahead for this.

$$LookAheadDist = \max(30, \text{round}(\frac{30 * BestDistanceFlown}{NumberOfPilotsLandedOut}, 0))$$

$$\forall i : i \leq \text{int}(MaxDist * 10) : Difficulty_i = \sum_{j=i}^{j=\min(i+LookAheadDist, \text{int}(BestDistanceFlown*10))} PilotsLanded_j$$

$$SumOfDifficulty = \sum_i Difficulty_i$$

Relative difficulty is then calculated by dividing each 100-meter slot's difficulty by twice the sum of all difficulty values.

$$\forall i : i \leq \text{int}(MaxDist * 10) : RelativeDifficulty_i = \frac{Difficulty_i}{2 * SumOfDifficulty}$$

Finally, we can calculate the difficulty score percentage for each 100-meter slot.

$$\forall i : i \leq \text{int}(MinDist * 10) : DiffScore_i = \sum_{j=0}^{j=\text{int}(MinDist*10)} RelativeDifficulty_j$$

$$\forall i : i > \text{int}(MinDist * 10) \& i < \text{int}(BestDistanceFlown * 10) : DiffScore_i = \sum_{j=0}^{j=i} RelativeDifficulty_j$$

$$\forall i : i \geq \text{int}(BestDistanceFlown * 10) : DiffScore_i = 0.5$$

 The difficulty calculation does not apply to paragliding.

#### 11.1.1.1 Example for difficulty calculation

 For an example of how the difficulty calculation works, see Figure 7: Note how the slope of the green curve (the total Distance points) becomes steeper before an area where many pilots landed and flatter just after. The red circles show these areas before the big group at the 41 km mark, and after the 46 km mark. There are two reasons for this:

For safety and retrieval reasons, we do not want to encourage pilots to fly only a short distance past a group of landed pilots.

If a pilot lands somewhere, he or she probably got into trouble just before, and then glided a while before landing.

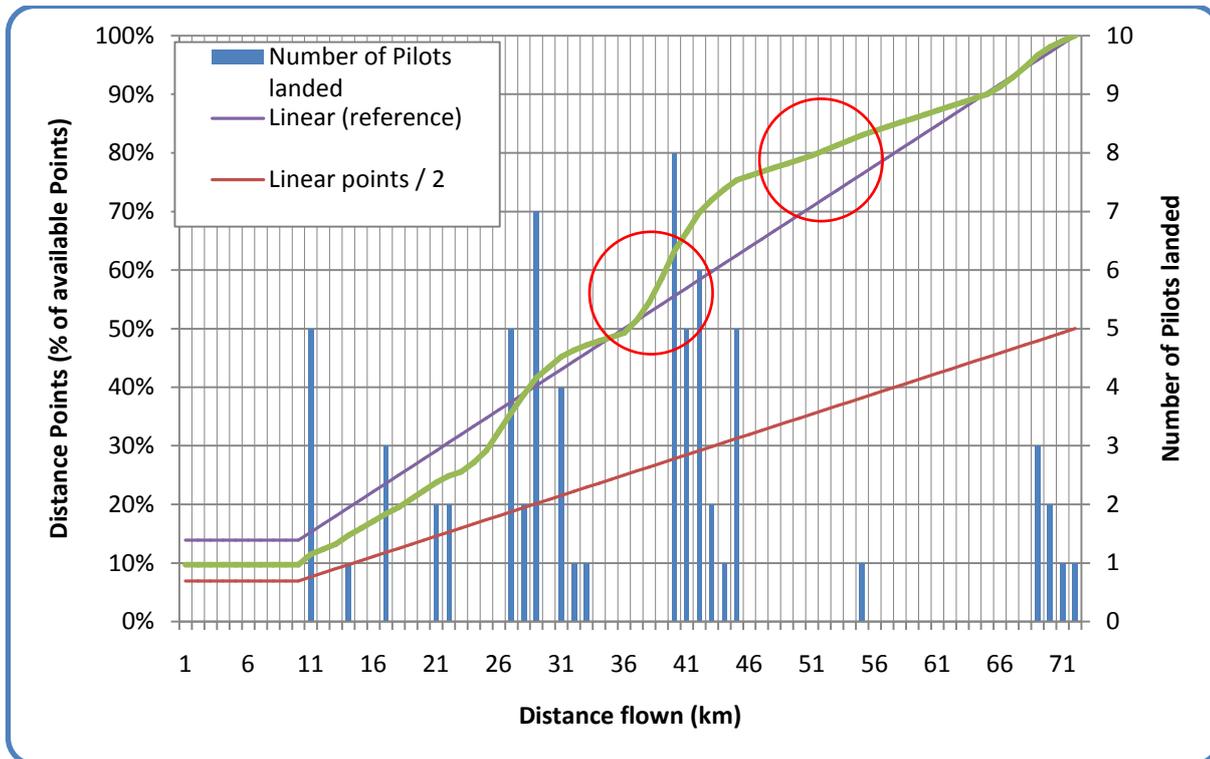


Figure 7: Sample Distance Points

## 11.2 Time points

Time points are assigned to the pilot as a function of best time and pilot time – the time the pilot took to complete the speed section. Slow pilots will get zero points for speed if their time to complete the speed section is equal to or longer than the fastest time plus the square root of the fastest time. All times are measured in hours.

$$SpeedFraction_p = \max\left(0, 1 - \frac{(Time_p - BestTime)^5}{\sqrt{BestTime}^6}\right)$$

$$TimePoints_p = SpeedFraction_p * AvailableTimePoints$$

### 11.2.1 Examples

For three examples of Time Point distributions for tasks with different best times, see Figure 8 and Table 1. The best time is defined as the time of the fastest pilot over the speed section who also reached the goal.

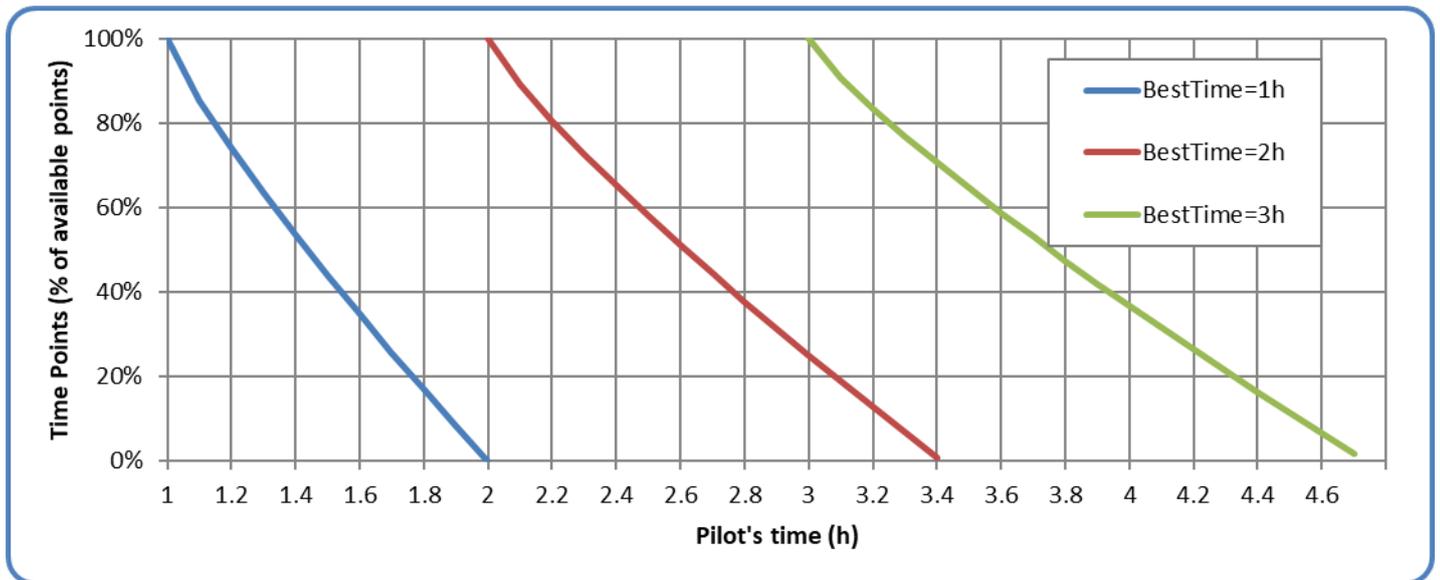


Figure 8: Sample time point distributions

Fastest Time	80% Time Points time	50% Time Points time	0 Time Points time
1:00	1:08:42	1:26:07	2:00:00
2:00	2:12:18	2:36:56	3:24:51 (3.41 hours)
3:00	3:15:04	3:45:14	4.43:55 (4.73 hours)

Table 1: Sample time point distribution (times in hours:minutes:seconds)

### 11.3 Leading points

Leading points are awarded to encourage pilots to start early and to reward the risk involved in flying in the leading group. Pilots will get leading points even if they landed before goal or the end of speed section.

$$LC_{\min} = \min(\forall p : p \in PilotsFlown : LC_p)$$

$$LeadingFactor_p = \max(0, 1 - \sqrt[3]{\frac{(LC_p - LC_{\min})^2}{\sqrt{LC_{\min}}}}$$

$$LeadingPoints_p = LeadingFactor_p * AvailableLeadingPoints$$

To get an impression of the way leading points are awarded depending on a task's minimal leading coefficient, see Figure 9.

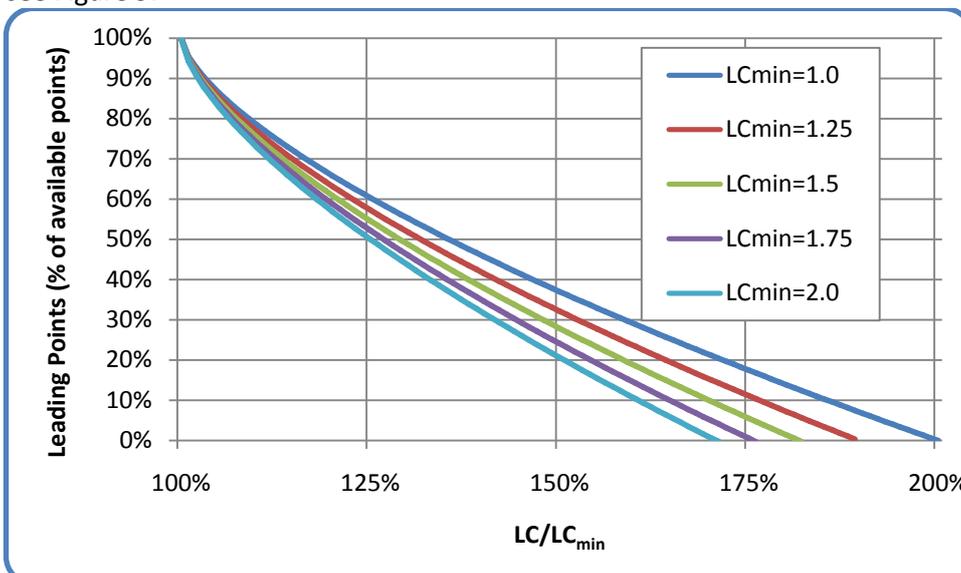


Figure 9: Leading points for various LC<sub>min</sub>

### 11.3.1 Leading coefficient

Each started pilot's track log is used to calculate the leading coefficient (LC), by calculating the area underneath a graph defined by each track point's time, and the distance to ESS at that time. The times used for this calculation are given in seconds from the moment when the first pilot crossed SSS, to the time when the last pilot reached ESS. For pilots who land out after the last pilot reached ESS, the calculation keeps going until they land. The distances used for the LC calculation are given in kilometers and are the distance from each point's position to ESS, starting from SSS, but never more than any previously reached distance. This means that the graph never "goes back": even if the pilot flies away from goal for a while, the corresponding points in the graph will use the previously reached best distance towards ESS.

Calculation of the leading coefficient (LC) for each pilot follows this formula:

$$LC_p = \frac{\sum_{i:tp_i \in \text{TrackPointsInSS}_p} \text{taskTime}(tp_i) * (\text{bestDistToESS}(tp_{i-1})^2 - \text{bestDistToESS}(tp_i)^2)}{1800 * \text{LengthOfSpeedSection}^2}$$

$$\forall p : p \in \text{PilotsLandedOut} \wedge \text{taskTime}(tp_{\max}) < \text{ESSTime}_{\text{LastPilotAtESS}} :$$

$$LC_p = LC_p + \text{LastTime}_{\text{LastPilotAtESS}} * \text{bestDistToESS}(tp_{\max})^2$$

$$\forall p : p \in \text{PilotsLandedOut} \wedge \text{taskTime}(tp_{\max}) \geq \text{ESSTime}_{\text{LastPilotAtESS}} :$$

$$LC_p = LC_p + \text{taskTime}(tp_{\max}) * \text{bestDistToESS}(tp_{\max})^2$$

$$\text{taskTime}(tp) = \min(\text{TaskDeadline}, \text{time}(tp))$$

$$\text{bestDistToESS}(tp_0) = \text{LengthOfSpeedSection}$$

$$\forall i : i > 0 \wedge tp_i \in \text{TrackPointsInSS}_p :$$

$$\text{bestDistToESS}(tp_i) = \min(\text{bestDistToESS}(tp_{i-1}), \text{LengthOfSpeedSection} - \text{distanceFlown}(tp_i))$$

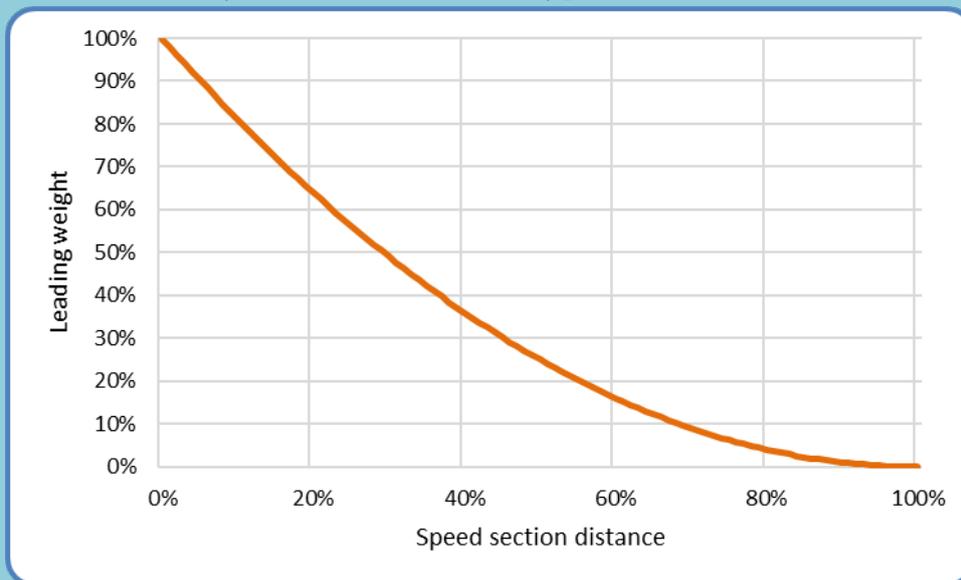


Figure 10: Implicit leading weight for hang-gliding

$$\text{leadingArea}_p =$$

$$\sum_{i:tp_i \in \text{trackPointsInSS}_p} \text{weight}(\text{minToESS}(tp_i)) * (\text{taskTime}(tp_i) - \text{taskTime}(tp_{i-1})) * \text{minToESS}(tp_i)$$

$$\text{bestTrackPoint}_p = \text{trackPointWithShortestDistanceToESS}(\text{trackPointsInSS}_p)$$

$$\text{missingArea}_p =$$

$$\text{weightFalling}(\text{distToEss}(\text{bestTrackPoint}_p)) * (\text{maxTime} - \text{bestTrackPoint}_p.\text{time}) *$$

$$\text{distToESS}(\text{bestTrackPoint}_p)$$

$$LC_p = \frac{\text{leadingArea}_p + \text{missingArea}_p}{1800 * \text{speedSectionDistance}}$$

$$\text{weight}(\text{missingDistance}) = \text{weightRising}(\text{missingDistance}) * \text{weightFalling}(\text{missingDistance})$$

$$\text{weightRising}(\text{missingDistance}) = \left(1 - 10^{9 \cdot \frac{\text{missingDistance}}{\text{speedSectionDistance}} - 9}\right)^5$$

$$\text{weightFalling}(\text{missingDistance}) = \left(1 - 10^{-3 \cdot \frac{\text{missingDistance}}{\text{speedSectionDistance}}}\right)^2$$

$$\text{minToESS}(tp_0) = \text{speedSectionDistance}$$

$$\forall i: i > 0, tp_i \in \text{trackPointsInSS}_p: \text{minToESS}(tp_i) = \min(\text{minToESS}(tp_{i-1}), \text{distToESS}(tp_i))$$

$$\text{taskTime}(\text{trackPoint}) = \min(\text{trackpoint.time}, \text{taskDeadline})$$

$$\text{maxTime} = \min(\max(\text{lastOutlandingTime}, \text{lastESStime}), \text{taskDeadline})$$

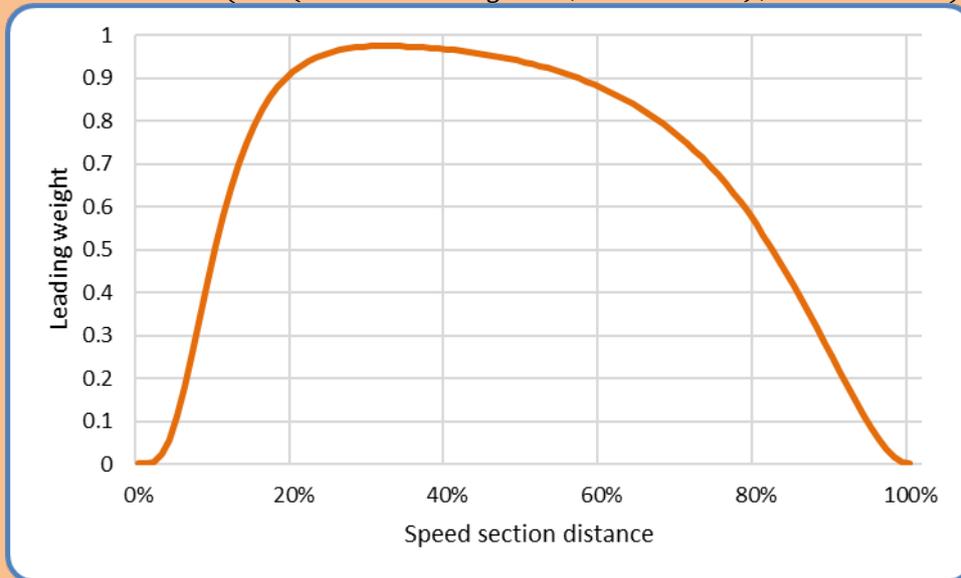


Figure 11: Leading weight for paragliding

### 11.3.2 Example

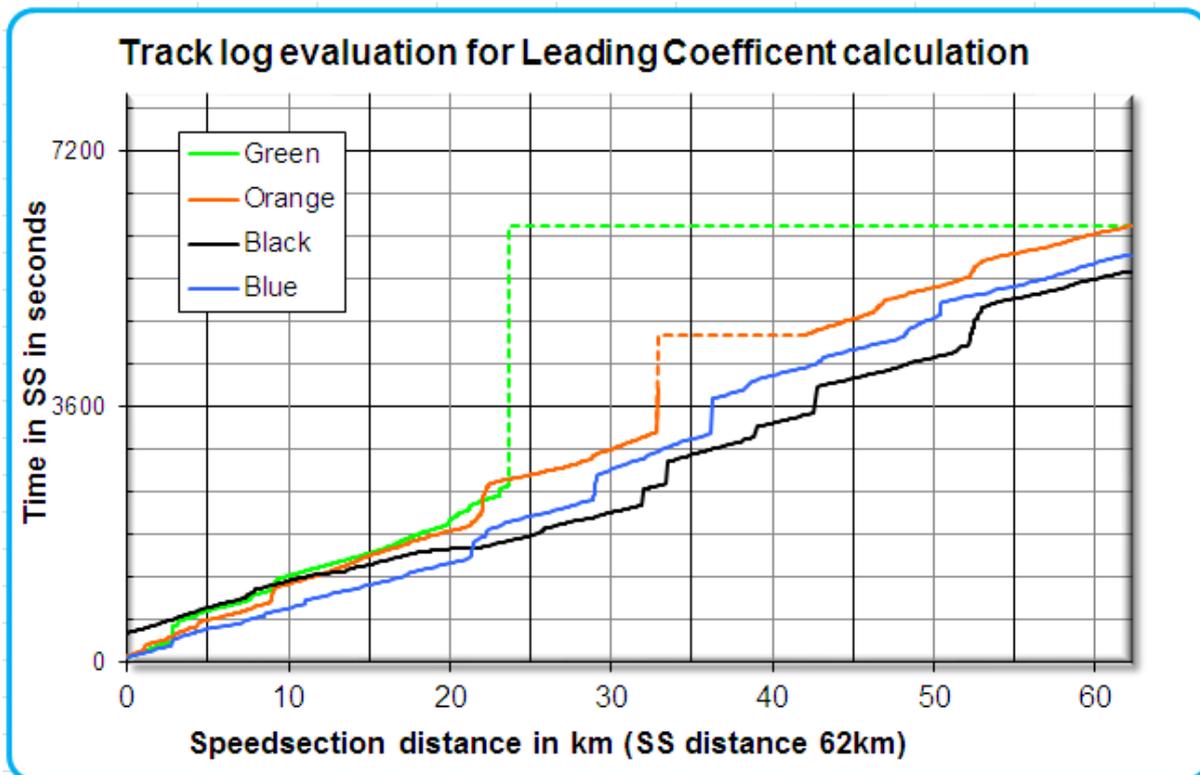


Figure 122: Sample track log graphs for LC calculation

Blue was the first to enter the speed section, but Black was the first pilot to cross the end of speed section. Green started at the same time as Blue, but landed short, after about 23km and just over 40 minutes of flight inside the speed section.

Black was fastest, therefore will get the most time points, but he started late, probably had pilots out front to show the way during the first 22km but was leading after that.

If a pilot lands along the course (Green), or if his track log is interrupted (Orange), his track log is completed as shown by the dotted lines: Missing parts are calculated as if the dotted line was the actual track log, so LC becomes bigger, lowering the leading points for that pilot, compared to a track where that part is not missing. A pilot landing just short of goal will be less penalised and could even get full leading points if he led for a long while.

The pilot who used best the earliest part of the day (i.e. Black, who has the smallest area below the track log graph) gets all the available leading points, while the others gets their points according to the same formula used for the time points for the same reasons. If the task in the example is fully valid, and 30% of pilots reached goal, then Black will get all of the available 81 leading points and full time points, as he was fastest; Blue gets 45 leading points because he started early but was slower; Orange receives only 18 leading points as he was slow and had a gap in his track log; Green gets 0 points even though he started early, because he was the slowest and landed fairly short.

### 11.4 Arrival points

Arrival points depend on the position at which a pilot crosses ESS: The first pilot completing the speed section receives the maximum available arrival points, while the others are awarded arrival points according to the number of pilots who reached ESS before them. The last pilot to reach ESS will always receive at least 20% of the available arrival points.

$$AC_p = 1 - \frac{PositionAtESS_p - 1}{NumberOfPilotsReachingESS}$$

$$ArrivalFraction_p = 0.2 + 0.037 * AC_p + 0.13 * AC_p^2 + 0.633 * AC_p^3$$

$$ArrivalPoints_p = ArrivalFraction_p * AvailableArrivalPoints$$

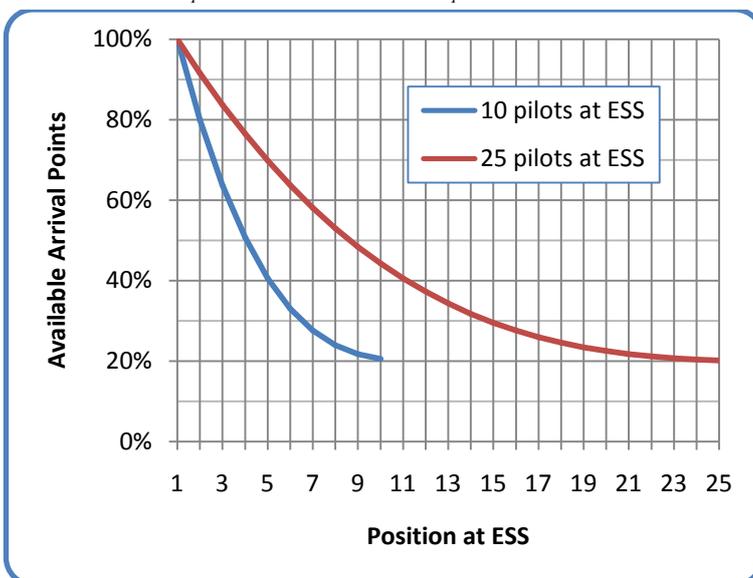


Figure13: Sample arrival points distributions

No arrival points are awarded in Paragliding.

## 12 Special cases

### 12.1 ESS but not goal

In a task where ESS and goal are not identical, a pilot may reach ESS, but not goal.

Reaching goal is seen as ‘validating’ one’s speed section performance. A pilot who does not reach goal after reaching ESS will lose a portion of his time points, as defined by the scoring system penalty parameter for this situation. He will also score full distance points for the distance covered and his full leading points. The timepoint penalty for not reaching goal is seen as a safety measure, since it encourages pilots to plan their final glide to ESS with enough altitude to safely reach goal.

For paragliders the scoring system parameter is to be set at 0% (i.e. no time points awarded) as this discourages high-speed final glides low to the ground.

For hang gliders the default scoring system parameter of 80% is recommended but can be changed by the local regulations to suit particular sites.

PG

$$\forall p : p \in \text{PilotsLandedBetweenESSandGoal} : \text{TotalScore}_p = \text{DistancePoints}_p + \text{LeadingPoints}_p + 0 * (\text{TimePoints}_p)$$

HG

$$\forall p : p \in \text{PilotsLandedBetweenESSandGoal} : \text{TotalScore}_p = \text{DistancePoints}_p + \text{LeadingPoints}_p + 0.8 * (\text{TimePoints}_p + \text{ArrivalPoints}_p)$$

### 12.2 Early start

An early start occurs if a pilot’s last SSS cylinder boundary crossing in start direction (enter or exit) occurred before the first (or only) start gate time.

In paragliding, pilots who perform an early start are only scored for the distance between the launch point and the SSS cylinder, as calculated when determining the complete task distance (see 6.5.1).

In hang-gliding, the so-called “Jump the Gun”-rule applies: If the early start occurred within a time that is close to the first (or only) start gate time, the pilot is scored for his complete flight, but a penalty is then applied to his total score.

The penalty calculation is based on two values X and Y, which are set in S7A, but can be changed at the task briefing (presumably by the meet director and/or the task committee). For each X seconds a pilot starts early, he incurs a 1 point penalty, up to a maximum of Y seconds. If a pilot starts more than Y seconds early, he will only be scored for minimum distance.

$$X_{\text{default}} = 2$$

$$Y_{\text{default}} = 300$$

$$\text{timeDiff}_p = \text{firstStartGateTime} - \text{lastStartTime}_p$$

$$\text{timeDiff}_p \leq 0 : \text{jumpTheGunPenalty}_p = 0$$

$$\text{timeDiff}_p > Y : \text{jumpTheGunPenalty}_p = 0, \text{totalScore}_p = \text{scoreForMinDistance}$$

$$0 < \text{timeDiff}_p \leq Y : \text{jumpTheGunPenalty}_p = \frac{\text{timeDiff}_p}{X}$$

$$\text{totalScore}_p = \max(\text{totalScore}_p - \text{jumpTheGunPenalty}_p, \text{scoreForMinDistance})$$

## 12.3 Stopped tasks

### 12.3.1 Stop task time

A task can be stopped at any time by the meet director. The time when a stop was announced for the first time is the “task stop announcement time”. This time must be recorded to score the task appropriately. For scoring purposes, a “task stop” time is calculated. This is the time which determines whether a task will be scored at all. Pilots’ flight will only be scored up to this task stop time.

 In hang-gliding, stopped tasks are “scored back” by a time that is determined by the number of start gates and the start gate interval: The task stop time is one start gate interval, or 15 minutes in case of a single start gate, before the task stop announcement time.

$numberOfStartGates = 1 : taskStopTime = taskStopAnnouncementTime - 15min.$

$numberOfStartGates > 1 : taskStopTime = taskStopAnnouncementTime - startGateInterval$

 In paragliding the score-back time is set as part of the competition parameters (see section 5.6).

$taskStopTime = taskStopAnnouncementTime - competitionScoreBackTime$

### 12.3.2 Requirements to score a stopped task

Stopped tasks will be scored only if they ran for a sufficiently long time.

$minimumTime = \min\left(1h, \frac{NominalTime}{2}\right)$

$typeOfTask = RaceToGoal \wedge numberOfStartGates = 1:$

$taskStopTime - startTime < minimumTime : taskValidity = 0$

$TypeOfTask \neq RaceToGoal \vee numberOfStartGates > 1:$

$taskStopTime - \max(\forall p: p \in StartedPilots: startTime_p) < minimumTime : taskValidity = 0$

### 12.3.3 Stopped task validity

For stopped tasks, an additional validity value, the Stopped Task Validity, is calculated and applied to the Task Validity.

  $DayQuality_{stopped} = LaunchValidity * DistanceValidity * TimeValidity * StoppedTaskValidity$

Stopped Task Validity is calculated taking into account the task distance, the flown distances of all pilots, the number of launched pilots and the number of pilots still flying at the time when the task was stopped.

$NumberOfPilotsReachedESS > 0 : StoppedTaskValidity = 1$

$NumberOfPilotsReachedESS = 0 :$

$StoppedTaskValidity = \min\left(1, \sqrt{\frac{BestDistFlown - \text{avg}(\forall i : DistFlown_i)}{DistLaunchToESS - BestDistFlown + 1}} * \sqrt{\frac{\text{stdev}(\forall i : DistFlown_i)}{5}}\right) + \left(\frac{NumPilotsLandedBeforeStopTime}{NumPilotsLaunched}\right)^3$

### 12.3.4 Scored time window

For stopped Race to Goal tasks with a single start gate, scoring considers the same time window for all pilots: The time between the race start and the task stop time.

$typeOfTask = RaceToGoal \wedge numberOfStartGates = 1 :$

$\forall p : p \in StartedPilots : scoreTimeWindow_p = (startTime, taskStopTime)$

For stopped Race to Goal tasks with multiple start gates, as well as Elapsed Time races, must be treated slightly differently: Only the time window available to the last pilot started is considered for scoring. This time window is defined as the amount of time  $t$  between the last pilot's start and the task stop time. For all pilots, only this time  $t$  after their respective start is considered for scoring.

$typeOfTask \neq RaceToGoal \vee numberOfStartGates > 1 :$

$scoreTime = taskStopTime - \max(\forall p : p \in StartedPilots : startTime_p)$

$\forall p : p \in StartedPilots : scoreTimeWindow_p = (startTime_p, startTime_p + scoreTime)$

This means that if the last pilot started and then flew for, for example, 75 minutes until the task was stopped, all tracks are only scored for the first 75 minutes each pilot flew after taking their respective start.

### 12.3.5 Time points for pilots at or after ESS

Pilots who were at a position between ESS and goal at the task stop time will be scored for their complete flight, including the portion flown after the task stop time. This is to remove any discontinuity between pilots just before goal and pilots who had just reached goal at task stop time.

A fixed amount of points is subtracted from the time points of each pilot that makes goal in a stopped task. This amount is the amount of time points a pilot would receive if he had reached ESS exactly at the task stop time. This is to remove any discontinuity between pilots just before ESS and pilots who had just reached ESS at task stop time.

$typeOfTask = RaceToGoal \wedge numberOfStartGates = 1 :$

$timePointsReduction = timePoints(taskStopTime - startTime)$

$TypeOfTask \neq RaceToGoal \vee numberOfStartGates > 1 :$

$timePointsReduction = timePoints(taskStopTime - \max(\forall p : p \in StartedPilots : startTime_p))$

$\forall p : p \in PilotsInGoal : finalTimePoints_p = timePoints_p - timePointsReduction$

### 12.3.6 Distance points with altitude bonus

To compensate for altitude differences at the time when a task is stopped, a bonus distance is calculated for each point in the pilots' track logs, based on that point's altitude above goal. This bonus distance is added to the distance achieved at that point. All altitude values used for this calculation are GPS altitude values, as received from the pilots' GPS devices (no compensation for different earth models applied by those devices). For all distance point calculations, including the difficulty calculations in hang-gliding (see 11.1.1), these new stopped distance values are being used to determine the pilots' best distance values. Time and leading point calculations remain the same: they are not affected by the altitude bonus or stopped distance values.

  $GlideRatio = 5.0$

  $GlideRatio = 4.0$

$\forall p : p \in PilotsLandedBeforeGoal : bestDistance_p = \max(\text{minimumDistance},$

$taskDistance - \min(\forall track_p : point_i :$

$\text{shortestDistanceToGoal}(track_p : point_i) - (track_p : point_i : altitude - GoalAltitude) * GlideRatio))$

$\forall p : p \in PilotsReachedGoal : bestDistance_p = taskDistance$

## 12.4 Penalties

Penalties for various actions are defined in the rules. These penalties are either expressed as an absolute number (e.g. "100 points") or as a percentage (e.g. "10% of the pilot's score in the task where he performed the

punishable action”). The corresponding number of points is then deducted from the punished pilot’s total score to calculate his final score.

$$finalScore_p = score_p - absolutePenalty_p$$

$$finalScore_p = score_p * (1 - percentagePenalty_p)$$

 These penalties are completely independent of any “Jump the Gun”-Penalty a pilot may have incurred.

The penalty mechanism can also be used to award bonus points to a pilot for some actions like helping a pilot in distress. In that case the penalty must be given as a negative number.

Any rounding up of scores is to be done after the application of penalties. The lowest score a pilot can attain in a task, regardless of any incurred penalties, is zero points.

## **13 Task ranking**

Definitions of types of task ranking are in Section 7A-5.2.4.

Overall task ranking

Female task ranking

Nation task ranking

## **14 Competitionranking**

Definitions of types of Competition ranking are in Section 7A-5.2.5.

Overall competition ranking

Female competition ranking

Nation competition ranking

Ties

## 15 FTV – Fixed Total Validity

Fixed Total Validity (FTV) is a procedure to score pilots on their best task performances, rather than all their tasks. Fixed Total Validity means the sum (total) of available points (validity) is set (fixed) to the same value for each competitor.

$$FTV\_factor = 0.2 \vee 0.25$$

$$CalculatedFTV = (1 - FTV\_factor) * \sum_{t:Task} \frac{WinnerScore_t}{1000}$$

To calculate a pilot's FTV score, for all his flights:

1. Calculate a performance percentage for each day by dividing the pilot's day score by the daywinner's points
2. Arrange all flights in descending order of performance percentage
3. Total up the flights' raw day scores (not performance percentages) in order of performance percentage until the sum of validities for those scores reaches the pre-decided Fixed Total Validity value.

If the last score added takes that pilot's total validity above the Fixed Total Validity, then only a fraction of that score is used so that the pilot's total validity is equal to the Fixed Total Validity.

$$\forall t : t \in ScoredTasks : Performance_{p,t} = \frac{Score_{p,t}}{WinnerScore_t}$$

$$SortedPerformance_p = \text{sortDescending}(\forall t : t \in ScoredTasks : Performance_{p,t})$$

$$OrderedValidities_p = \text{orderByPerformance}(SortedPerformance_p,$$

$$\forall t : t \in ScoredTasks : \frac{WinnerScore_t}{1000})$$

$$OrderedScores_p = \text{orderByPerformance}(SortedPerformance_p, \forall t : t \in ScoredTasks : Score_{p,t})$$

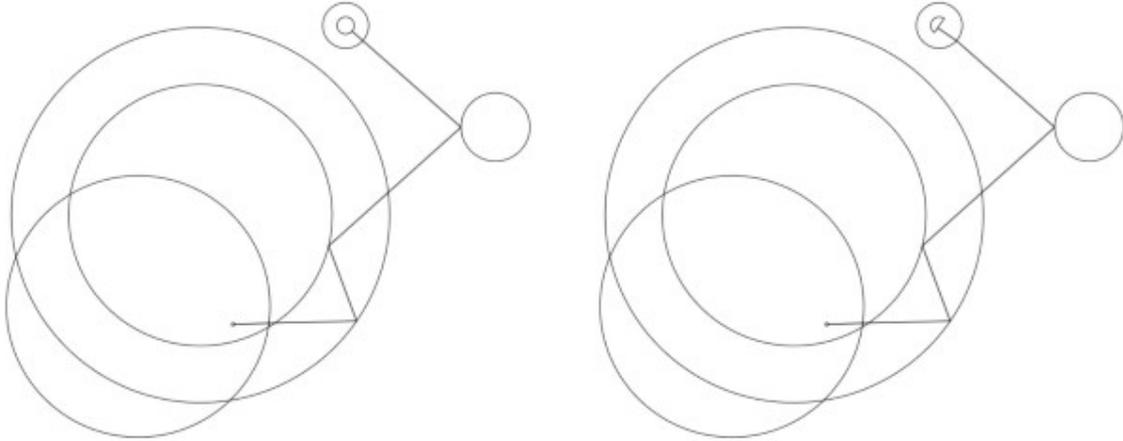
$$OrderedScores_{p,u} *$$

$$FTV\_Score_p = \sum_{u=0}^{numberOfTasks} \frac{\min(0, CalculatedFTV - (u > 0 : \sum_{v=0}^{\min(0,u-1)} OrderedValidities_{p,v}))}{\max(1, OrderedValidities_{p,u})}$$

## 16 Annexe A: Algorithm for finding shortest path

### 1 Introduction

Distance calculation in paragliding and hang-gliding races is based on finding the shortest path between a given start point and a goal (cylinder or goal line), touching any number of intermittent cylinders. This calculation can then be used to determine the distance of a set task, but also for the distance a pilot still must fly to complete a task.



*Shortest path to a goal cylinder (left) and a goal line (right). The effect of measuring the shortest path to the end of the speed section can be seen by the kinked leg to the goal line.*

While finding the shortest path seems to be simple for human beings using pen and paper, there exists no closed mathematical solution for this problem. We therefore must resort to an iterative approach that converges on a minimum in most cases.

The calculations required to determine this path are best suited to plane geometry, so all WGS84 coordinates, of starting point, of the turnpoint cylinder centers and goal points, are transformed to cartesian coordinates. The planar distance of the calculated path is not used, but the resulting position fixes on the cylinders (and goal line) are transformed back to WGS84 coordinates and used to calculate the ellipsoidal distance.

This document describes the algorithms needed to perform these calculations, which are as much about speed and ease of use as they are about acceptable accuracy.

What this document does not yet address is:

- the transformation method from WGS84 to cartesian coordinates
- the method of transforming cartesian fix positions back to WGS84 coordinates

### 2 Calculations

These calculations require a set of points representing the cylinders, comprising a center position in cartesian coordinates, a radius in metres and a fix position that will mark the point on the cylinder (or line) of the shortest path. This fix position is initially set as the center position.

When the calculations are used to determine the distance a pilot still must fly to goal, the set of points is created from the pilot's track point and the remaining cylinders not yet reached by the pilot.

#### 2.1 Core algorithm

The base calculation uses three sequential points (P1, P2, P3) and finds the shortest distance between P1 and P3 that touches the P2 cylinder. P1 and P3 are taken from their fix positions. The

point found on the P2 cylinder is stored as the P2 fix. The leg distance is measured between the P1 and P2 fix positions.

The algorithm sequentially steps through the remaining points and performs this calculation on each one, so the most recent P2 fix becomes the P1 fix used in the next step. On completion, all points will have a fix position on their cylinder (or line) and the shortest distance found will be the sum of the leg distances.

This algorithm is then repeated, using the points from the previous iteration, until the difference between the current shortest distance and the last shortest distance is either less than a certain tolerance, or the maximum number of iterations has been reached.

## **2.2 Required settings**

In order that implementations give the same results, it is necessary to define some basic settings.

### *Tolerance*

The value of the tolerance affects the final task distance. It also affects the number of iterations required, which itself is dependent on the complexity of the task.

A tolerance of 1 metre is enough, since all other distance measurements are also in meters. A smaller value may shorten the planar distance by a fraction of one metre, but this will not always be reflected in the final ellipsoidal distance and will require many more additional iterations.

### *Maximum number of iterations*

The algorithm does not in all cases converge on the absolute minimum distance. To ensure completion, and prevent infinite loops, we need to define a maximum number of iterations, to cover the case where a local minimum is encountered, and a subsequent iteration results in a distance larger than the previous one is found. This could happen in complex cylinder configurations or if a cylinder erroneously intersects another. The maximum number of iterations is set to 200.

### *Order of evaluation*

The first point used in the evaluation must be at index 1, because the distance is measured from the center of the launch waypoint (regardless of whether it has been given a radius).

The order in which the points are evaluated is also important. While the number of iterations may be reduced by evaluating odd then even indexed points, or other such tricks, the results cannot be consistently replicated across different cylinder configurations. Evaluation order must be sequentially upwards.

### *Summary of required settings:*

- The tolerance value is 1 metre.
- The maximum number of iterations is 200.
- Points are evaluated sequentially upwards.

### 3 Definitions

The pseudo code used to convey the intention and flow of the calculations is written in a C-like style and presented as a series of simple functions.

#### 3.1 *Point object*

In these examples the object, or struct, representing a cylinder or line is denoted as a `point`. It has the following properties, or members:

**x** (float) the x coordinate

**y** (float) the y coordinate

**radius** (int) the radius in metres

**fx** (float) the x coordinate of the fix

**fy** (float) the y coordinate of the fix

On construction, or when creating a new instance, the `fx` and `fy` properties must be initialized to the `x` and `y` values.

```
functioncreatePoint(x,y,radius=0)
{
  //UsingECMAScriptobjectliteralstoconveyobjectcreation
  return{x:x,y:y,radius:radius,fx:x,fy:y};
}

functioncreatePointFromCenter(point)
{
  returncreatePoint(point.x,point.y,point.radius)
}

functioncreatePointFromFix(point)
{
  returncreatePoint(point.fx,point.fy,point.radius)
}
```

#### 3.2 *Planar calculations*

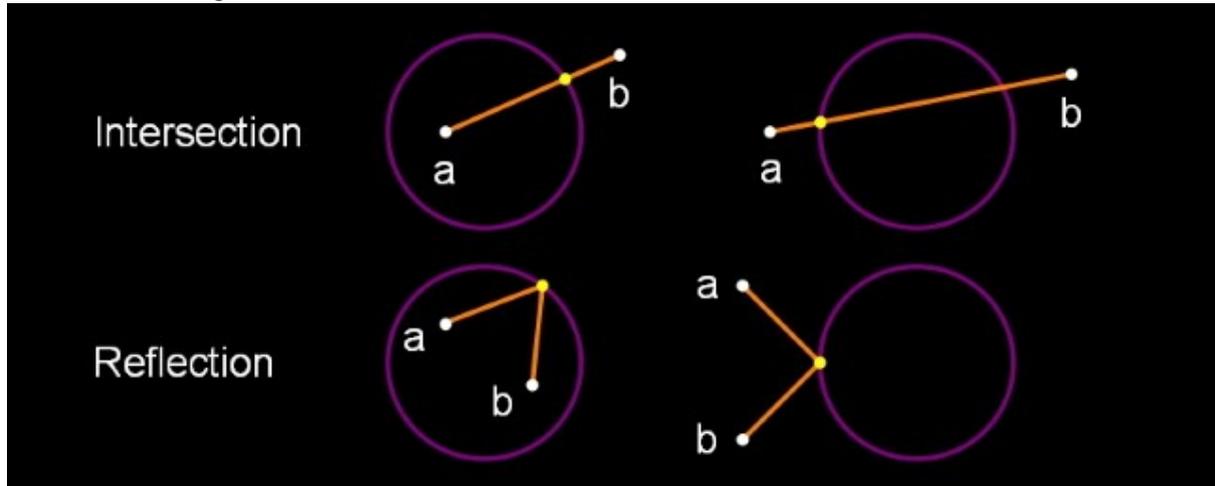
The entry point is the `getShortestPath` function, which initiates and keeps track of the distances found and the number of iterations.

The main function is `optimizePath`, which is responsible for stepping through the set of points and calculating the shortest distance. It uses the following helper functions:

- `getTargetPoints`
- `processCylinder`
- `processLine`

### 3.2.1 Cylinder evaluation

In general the solution to finding the shortest path is either one of intersection or reflection, as shown in the image below.



*The yellow dot marks the fix on the cylinder between point A and point B.*

The functions to solve these problems are:

- [setIntersection1](#)
- [setIntersection2](#)
- [setReflection](#)

Exceptions to the above are when point A and point B are the same, or when one them is on the cylinder. The latter situation can only occur if a cylinder erroneously intersects another, which may result in a miscalculation or extra iterations. These situations are handled by:

- [projectOnCircle](#)
- [pointOnCircle](#)

### 3.2.2 getShortestPath

This is the outer controlling function that iterates through a sequence of distance optimizations until the difference between the optimized distances obtained by the previous and the current iteration is less than the tolerance, or the maximum number of iterations has been reached.

```
//Inputs:
//points-arrayofpointobjects
//line-goallineendpoints,oremptyarray

functiongetShortestPath(points,line)
{
tolerance=1.0;
lastDistance=INT_MAX;
finished=false;
count=getArrayLength(points);

//opsCountisthenumberofiterationsallowed
opsCount=200;

while(!finished&&opsCount-->0){
distance=optimizePath(points,count,line);

//Seeifthedifferencebetweenthelastdistanceis
//smallerthanthetolerance
finished=lastDistance-distance<tolerance;
lastDistance=distance;
}
}
```

### 3.2.3 optimizePath

The algorithm sequentially steps through the cylinder points, taking three consecutive points at each step, with the middle one being the target of the calculation.

Each set of three points is passed to an appropriate function that finds the shortest path between the outer points and the target. The position of the fix on the target cylinder (or goal line) is set as a property of the target point. The target point subsequently becomes the first point in the next iteration step and this fix property is used to denote its position.

```
//Inputs:
//points-arrayofpointobjects
//count-numberofpoints
//line-goallineendpoints,oremptyarray

functionoptimizePath(points,count,line)
{
distance=0;
hasLine=getArrayLength(line)==2;

for(index=1;index<count;index++){
//Getthetargetcylindercanditsprecedingandsucceedingpoints
c,a,b=getTargetPoints(points,count,index);

if(index==count-1&&hasLine){
processLine(line,c,a);
}else{
processCylinder(c,a,b);
}

//CalculatethedistancefromAtotheCfixpoint
legDistance=hypot(a.x-c.fx,a.y-c.fy);
distance+=legDistance;
}

returndistance;
}
```

### 3.2.4 getTargetPoints

Returns a set of three consecutive points comprising the target cylinder C, plus its preceding and succeeding points (A and B). The target point C is taken directly from the points array (ie. it is a reference, or copied by reference), while the other two are created as new point objects from either their fix or center positions.

The end of speed section is taken from its center position, so that its fix is pinned to the preceding points, rather than any subsequent points at a different position.

```
//Inputs:
//points-arrayofpointobjects
//count-numberofpoints
//index-indexofthetargetcylinder(from1upwards)

functiongetTargetPoints(points,count,index)
{
//SetpointCtothetargetcylinder
c=points[index];

//CreatepointAusingthefixfromthepreviouspoint
a=createPointFromFix(points[index-1]);

//CreatepointBusingthefixfromthenextpoint
//(usepointCcenterforthelastpoint).

if(index==count-1){
b=createPointFromCenter(c);
}else{
b=createPointFromFix(points[index+1]);
}

return[c,a,b];
}
```

### 3.2.5 processCylinder

Sets the fix on the target cylinder C from the fixes on the previous point A and the next point B. The distance from point C center to the AB line segment determines if it intersects the cylinder ( $\text{distCtoAB} < C$  radius). If it does and both points are inside the cylinder it requires the reflection solution, otherwise one of the intersection solutions. If the line does not intersect the cylinder or is tangent to it, it requires the reflection solution.

See [Cylinder evaluation](#) for more information.

```
//Inputs:
//c,a,b-targetcylinder,previouspoint,nextpoint

functionprocessCylinder(c,a,b)
{
  distAC,distBC,distAB,distCtoAB=getRelativeDistances(c,a,b);

  if(distAB==0.0){
    //AandBarethesamepoint:projectthepointonthecircle
    projectOnCircle(c,a.x,a.y,distAC);

  }elseif(pointOnCircle(c,a,b,distAC,distBC,distAB,distCtoAB){
    //AorBareonthecircle:thefixhasbeencalculated
    return;

  }elseif(distCtoAB<c.radius){
    //ABsegmentintersectsthecircle,butisnottangenttoit

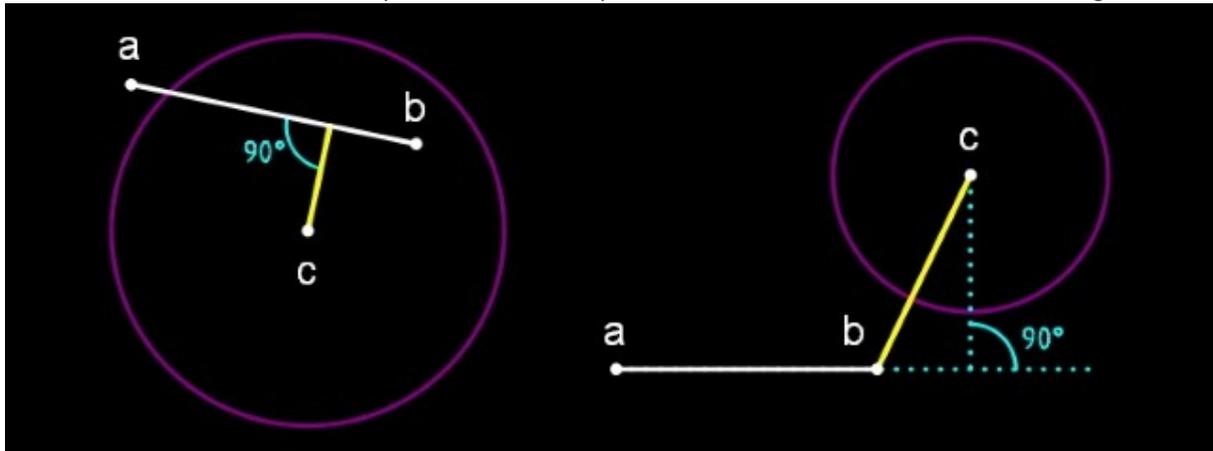
    if(distAC<c.radius&&distBC<c.radius){
      //AandBareinsidethecircle
      setReflection(c,a,b);

    }elseif(distAC<c.radius&&distBC>c.radius ||
      (distAC>c.radius&&distBC<c.radius)){
      //Onepointinside,onepointoutsidethecircle
      setIntersection1(c,a,b,distAB);

    }elseif(distAC>c.radius&&distBC>c.radius){
      //AandBareoutsidethecircle
      setIntersection2(c,a,b,distAB);
    }
  }else{
    //AandBareoutsidethecircleandtheABsegmentis
    //eithertangenttoitorordoesnotintersectit
    setReflection(c,a,b);
  }
}
```

### 3.2.6 getRelativeDistances

Returns the distances between points A, B and C, plus the distance from C to the AB line segment.



The distance from C to the AB line segment is shown by the yellow line.

```
//Inputs:
//c,a,b-targetcylinder,previouspoint,nextpoint

functiongetRelativeDistances(c,a,b)
{
//CalculatedistancesAC,BCandAB
distAC=hypot(a.x-c.x,a.y-c.y);
distBC=hypot(b.x-c.x,b.y-c.y);
len2=(a.x-b.x)**2+(a.y-b.y)**2;
distAB=sqrt(len2);

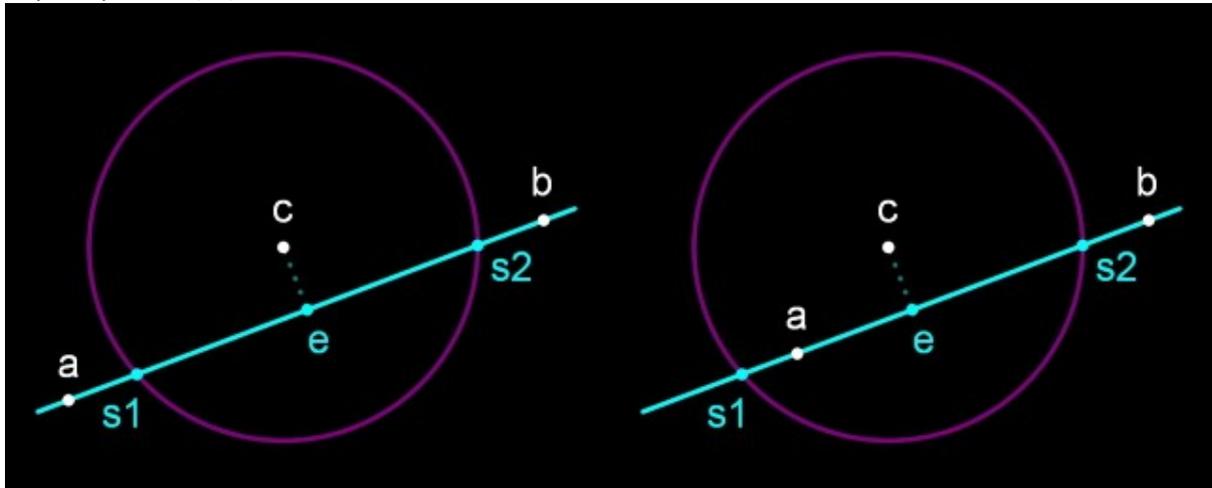
//FindtheshortestdistancefromCtotheABlinegment
if(len2==0.0){
//AandBarethesamepoint
distCtoAB=distAC;
}else{
t=((c.x-a.x)*(b.x-a.x)+(c.y-a.y)*(b.y-a.y))/len2;

if(t<0.0){
//BeyondtheAendoftheABsegment
distCtoAB=distAC;
}elseif(t>1.0){
//BeyondtheBendoftheABsegment
distCtoAB=distBC;
}else{
//OntheABsegment
cpx=t*(b.x-a.x)+a.x;
cpy=t*(b.y-a.y)+a.y;
distCtoAB=hypot(cpx-c.x,cpy-c.y);
}
}

return[distAC,distBC,distAB,distCtoAB];
}
```

### 3.2.7 getIntersectionPoints

Returns the two intersection points (s1, s2) of circle C by the AB line, plus the midpoint (e) of the line between them. The intersection points will either be within the AB segment, beyond point A (s1) or beyond point B (s2).



```
//Inputs:
//c,a,b-targetcylinder,previouspoint,nextpoint
//distAB-ABlinesegmentlength

functiongetIntersectionPoints(c,a,b,distAB)
{
//Finde,whichisontheABlineperpendiculartocenter
dx=(b.x-a.x)/distAB;
dy=(b.y-a.y)/distAB;
t2=dx*(c.x-a.x)+dy*(c.y-a.y);

ex=t2*dx+a.x;
ey=t2*dy+a.y;

//Calculatetheintersectionpoints,s1ands2
dt2=c.radius**2-(ex-c.x)**2-(ey-c.y)**2;
dt=dt2>0?sqrt(dt2):0;

s1x=(t2-dt)*dx+a.x;
s1y=(t2-dt)*dy+a.y;
s2x=(t2+dt)*dx+a.x;
s2y=(t2+dt)*dy+a.y;

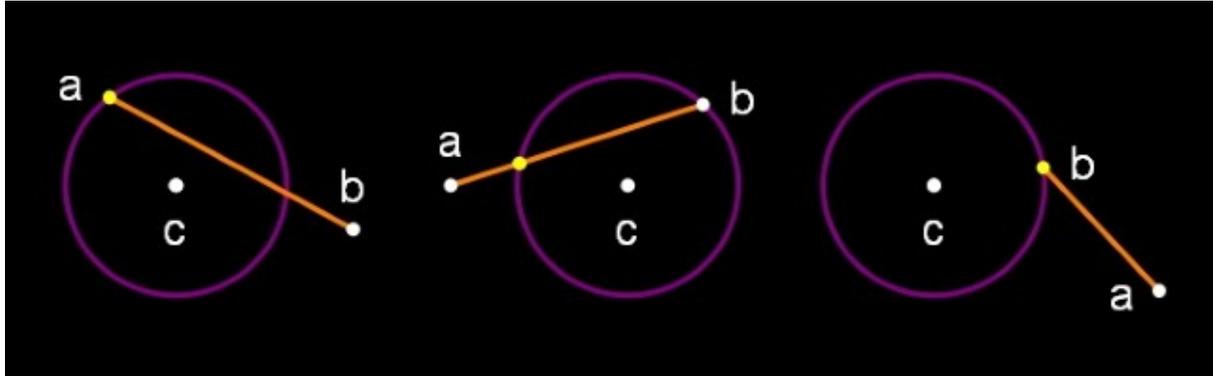
return[createPoint(s1x,s1y),createPoint(s2x,s2y),createPoint(ex,ey)];
}
```

### 3.2.8 pointOnCircle

Sets the C fix position if either point A or point B is on the circle C.

- If point A is on the circle, the fix is taken from point A.
- If point B is on the circle, the fix is either taken from the intersection point closest to point A (if the AB segment intersects the circle and point A is outside it), or from point B.

Returns **false** if neither point is on the circle, otherwise **true**.



```
//Inputs:
//c,a,b-targetcylinder,previouspoint,nextpoint
//Distancesbetweenthepoints

functionpointOnCircle(c,a,b,distAC,distBC,distAB,distCtoAB)
{
  if(fabs(distAC-c.radius)<0.0001){
    //Aonthecircle(perhapsBaswell):useAposition
    c.fx=a.x;
    c.fy=a.y;
    returntrue;
  }

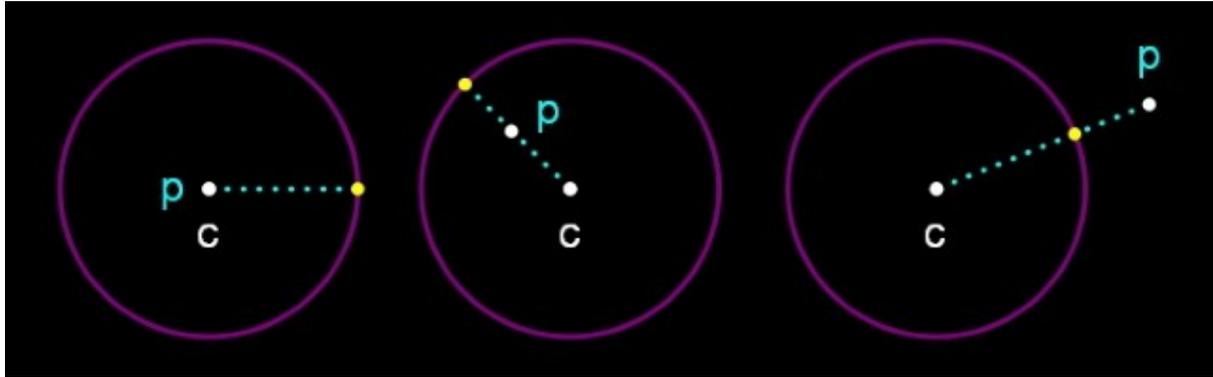
  if(fabs(distBC-c.radius)<0.0001){
    //Bonthecircle

    if(distCtoAB<c.radius&&distAC>c.radius){
      //ABsegmentintersectsthecircleandAisoutsideit
      setIntersection2(c,a,b,distAB);
    }else{
      //UseBposition
      c.fx=b.x;
      c.fy=b.y;
    }
    returntrue;
  }

  returnfalse;
}
```

### 3.2.9 projectOnCircle

Projects a point (P) on the circle C, at the intersection of the circle by the CP line. The result is stored in the C fix position.

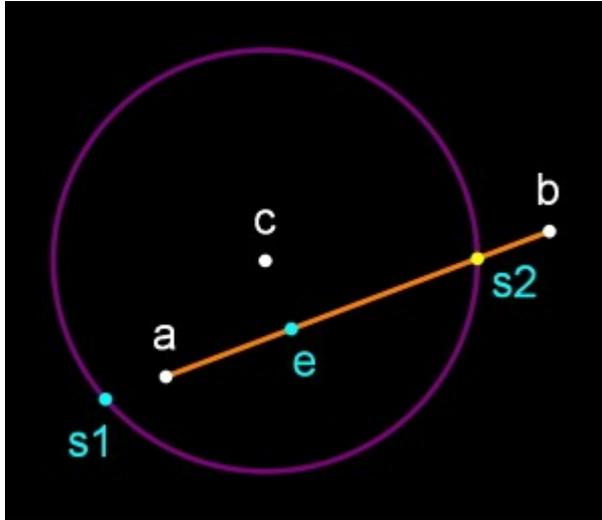


```
//Inputs:
//c-thecircle
//x,y-coordinatesofthepointtoproject
//len-linesegmentlength,fromctothe point

functionprojectOnCircle(c,x,y,len)
{
  if(len==0.0){
    //Thedefaultdirectioniseastwards(90degrees)
    c.fx=c.radius+c.x;
    c.fy=c.y;
  }else{
    c.fx=c.radius*(x-c.x)/len+c.x;
    c.fy=c.radius*(y-c.y)/len+c.y;
  }
}
```

### 3.2.10 setIntersection1

Sets the intersection of circle C by the AB line segment when one point is inside and the other is outside the circle (ie. there is only one intersection point on the AB line segment). The result is stored in the C fix position.



```
//Inputs:
//c,a,b-targetcylinder,previouspoint,nextpoint
//distAB-ABlinesegmentlength

functionsetIntersection1(c,a,b,distAB)
{
//Gettheintersectionpoints(s1,s2)
s1,s2,e=getIntersectionPoints(c,a,b,distAB);

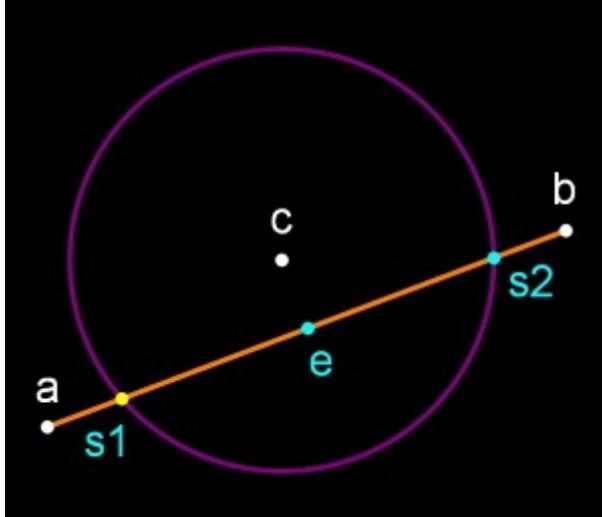
as1=hypot(a.x-s1.x,a.y-s1.y);
bs1=hypot(b.x-s1.x,b.y-s1.y);

//Findtheintersectionlyingbetweenpointsaandb
if(fabs(as1+bs1-distAB)<0.0001){
c.fx=s1.x;
c.fy=s1.y;
}else{
c.fx=s2.x;
c.fy=s2.y;
}
}
```

### 3.2.11 setIntersection2

Sets the intersection of circle C by the AB line segment when both points are outside the circle (ie. there are two intersection points on the AB line segment).

The result is stored in the C fix position and is the intersection that is closest to point A. This will lie between point A and point E (the midpoint of the line between the intersection points).



```
//Inputs:
//c,a,b-targetcylinder,previouspoint,nextpoint
//distAB-ABLineselementlength

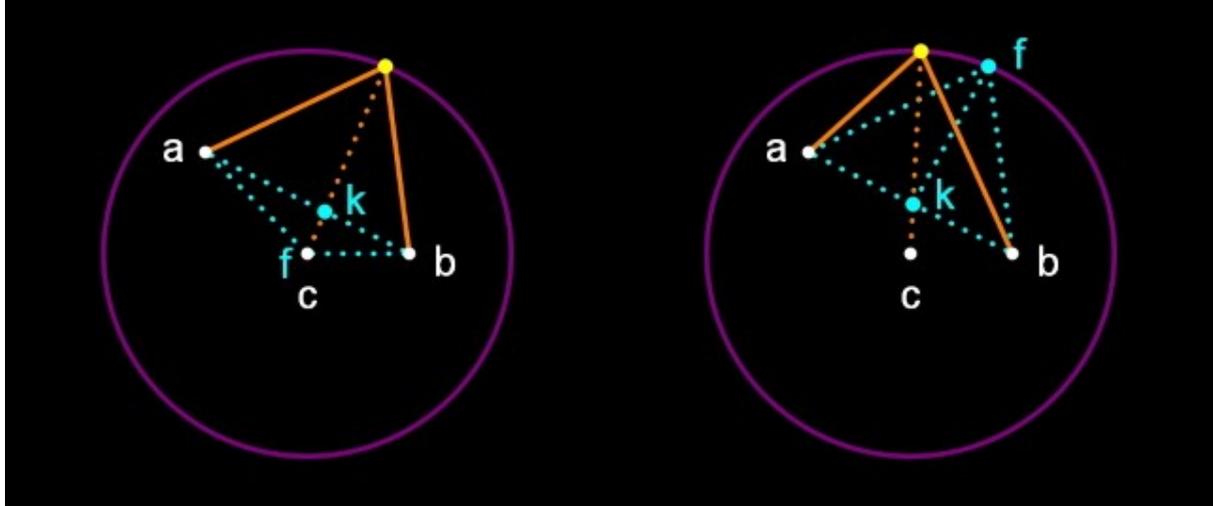
functionsetIntersection2(c,a,b,distAB)
{
//Gettheintersectionpoints(s1,s2)andmidpoint(e)
s1,s2,e=getIntersectionPoints(c,a,b,distAB);

as1=hypot(a.x-s1.x,a.y-s1.y);
es1=hypot(e.x-s1.x,e.y-s1.y);
ae=hypot(a.x-e.x,a.y-e.y);

//Findtheintersectionbetweenpointsaande
if(fabs(as1+es1-ae)<0.0001){
c.fx=s1.x;
c.fy=s1.y;
}else{
c.fx=s2.x;
c.fy=s2.y;
}
}
```

### 3.2.12 setReflection

Sets the reflection point of two external or internal points (A and B) on the circle C. This uses the triangle AFB (where F is the current C fix position) to find the point (K) on the AB line segment where it is cut by the AFB angle bisector. The reflected point is found by projecting K on the circle and is stored as the latest C fix position.



```
//Inputs:
//c,a,b-targetcircle,previouspoint,nextpoint

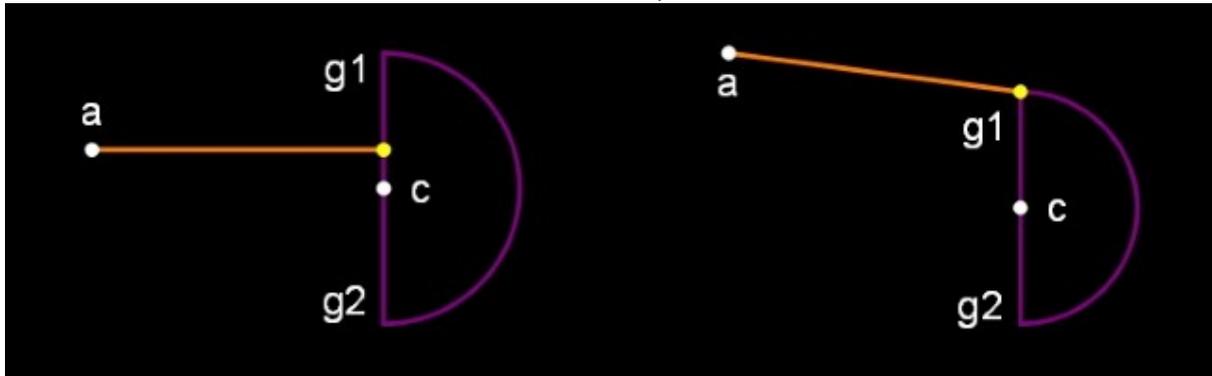
functionsetReflection(c,a,b)
{
//Thelengthsoftheadjacenttrianglesides(af,bf)are
//proportionaltothelengthsofthecutABsegments(ak,bk)
af=hypot(a.x-c.fx,a.y-c.fy);
bf=hypot(b.x-c.fx,b.y-c.fy);
t=af/(af+bf);

//CalculatepointkontheABsegment
kx=t*(b.x-a.x)+a.x;
ky=t*(b.y-a.y)+a.y;
kc=hypot(kx-c.x,ky-c.y);

//Projectkontotheradiusofc
projectOnCircle(c,kx,ky,kc);
}
```

### 3.2.13 processLine

Finds the closest point on the goal line (g1, g2) from point A, storing the result in the C fix position. This will either be on the line itself, or at one of its endpoints.



```
//Inputs:
//line-arrayofgoallineendpoints
//c,a-target(goal),previouspoint

functionprocessLine(line,c,a)
{
g1=line[0],g2=line[1];
len2=(g1.x-g2.x)**2+(g1.y-g2.y)**2;

if(len2==0.0){
//Errortrapping:g1andg2arethesamepoint
c.fx=g1.x;
c.fy=g1.y;
}else{
t=((a.x-g1.x)*(g2.x-g1.x)+(a.y-g1.y)*(g2.y-g1.y))/len2;

if(t<0.0){
//Beyondtheg1endofthelinesegment
c.fx=g1.x;
c.fy=g1.y;
}elseif(t>1.0){
//Beyondtheg2endofthelinesegment
c.fx=g2.x;
c.fy=g2.y;
}else{
//Projectionfallsonthelinesegment
c.fx=t*(g2.x-g1.x)+g1.x;
c.fy=t*(g2.y-g1.y)+g1.y;
}
}
}
```